

# Homework 4

- On all of your homework, you are welcome to use anything that we proved in class to help you solve the problems. You may also use any of the earlier homework problems.

1. Use induction or strong induction to prove that  $3 \mid (4^n - 1)$  for every  $n \in \mathbb{N}$ .
2. Prove that the sum of the first  $n$  odd natural numbers equals  $n^2$  by induction or strong induction.
3. Make a conjecture as to which  $n \in \mathbb{N}$  have the property that  $n! > 2^n$ . Then prove your conjecture by induction or strong induction.
4. In this exercise you will use strong induction to study sequences which are defined *recursively*.
  - (a) Define a sequence  $a_1, a_2, a_3, \dots$  recursively where  $a_1 = 1$ ,  $a_2 = 3$ , and for  $n \geq 3$ ,  $a_n = 2a_{n-1} - a_{n-2}$ . Prove that  $a_n = 2n - 1$  for all  $n \in \mathbb{N}$ .
  - (b) Define a sequence  $a_1, a_2, a_3, \dots$  recursively where  $a_1 = 1$ ,  $a_2 = 4$ , and for  $n \geq 3$ ,  $a_n = 2a_{n-1} - a_{n-2} + 2$ . Through scratch work, conjecture a formula for  $a_n$ , and then prove that your conjecture is correct.
5. Explain the error in the “proof” of Fake Proposition 4.12.

