PRACTICE SET FOR MIDTERM 2

A)Carefully study the examples solved in class.

B)Do the practice problems from sections :
2.7 (Related Rates)
3.7 (L'Hospital's rule)
4.1 (Max/Min)
4.2 (Theorems about differentiable functions)
4.4 (Curve Sketching)
4.5 (Optimization Problems)

C) Do the following equations admit any real solutions? If so, how many?

$$1) \; x^5 + \frac{1}{3}x^3 = 3 - e^x$$

$$2) \; 2x^5 + 5x^4 - 3 = 0$$

$$3*) \; \text{arctg}(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{25}{12}$$

$$4*) \; x^2 = \sin(x)$$

D) Prove that $\ln(x+1) \le x$ for every $x > -1$.

E) Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ is a function such that $f'(x) < 0 \quad \forall x \in \mathbb{R}$ and such that $\lim_{x \to -\infty} f(x) = +\infty$. Is it true that the equation $f(x) = 0$ has exactly one real solution?

F*) Can you find a differentiable function $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that f(5)=5, f(-5)=-5 and $f'(x) \ge x^2 + 2$?

G*) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be an even and differentiable function. Assuming the derivative is a continuous function, compute

$$\lim_{x \to 0} \frac{f(x) - f(0)}{\sin(x)}.$$

H*) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$. Suppose that $f(0) = 1$, $f'(0) = 5$ and $f''(x) < 0$ for every $x \in \mathbb{R}$. Prove that $f(x) \le 5x + 1$ for every $x \in \mathbb{R}$.