PRACTICE SET (PART 2)

Problem 1

1) Find an equation of the line passing through the origin and parallel to the tangent to the curve $x = t^2 + t$, $y = t^3 - 1$ at the point $(2, 0)$.
2) Find a parametrization of the circle with center $C = (1, 0)$ and radius 3 starting from $(1, 3)$ and going anticlockwise. Find an equation of the tangent at the point $(4, 0)$.
3) Find a parametrization of the ellipse centered at the origin with horizontal semiaxis 2 and vertical semiaxis 3 in anticlockwise direction starting from $(2, 0)$.
4) Find the length of the curve $x = e^{2t}\sin(2t)$, $y = e^{2t}\cos(2t)$ for $0 \leq t \leq \pi$.

Problem 2

1) Find the area enclosed by one petal of the following curves

$a) r = \cos(3\theta)$

$b) r = \sin(4\theta)$

2) Find the area of the region outside the curve $r = 2 + 2\sin\theta$ and inside the curve $r = 6\sin\theta$

Problem 3

Determine if the following sequences have a limit and in this case compute it.

(1) $a_n = \left(1 - \dfrac{1}{2n}\right)^n$

(2) $a_n = \left(\dfrac{n+3}{n}\right)^{2n}$

(3) $a_n = \left(\dfrac{n}{n+1}\right)^n$

(4) $a_n = \left(\dfrac{1}{n^2}\right)^{\frac{1}{n}}$

(5) $a_n = \sqrt{n}\log n$

(6) $a_n = \dfrac{2n-3}{n+2}\sin n$

(7) $a_n = \dfrac{\sin\left(\frac{1}{n}\right)}{n}$

(8) $a_n = n\sin\left(\dfrac{1}{n}\right)$

(9) $a_n = (-1)^n \dfrac{\sqrt{n}}{1-n}$

(10) $a_n = (-1)^n \left(1 + \dfrac{2}{n}\right)^n$

(11) $a_n = \dfrac{n^2+1}{3n^3-n}$

(12) $a_n = \dfrac{\sqrt[3]{5n+1}}{\sqrt[6]{n^2+n}}$

Problem 4
Study the convergence of the following series (absolute-conditional convergence, divergence)

(13) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{3n^5+n}}{n^2+\sqrt{n}}$

(14) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^6+3n}}{3n+n^3}$

(15) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{\sqrt{n}+1}}{n\sqrt{n}}$

(16) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\log^3 n}$

(17) $\displaystyle\sum_{n=1}^{\infty}\frac{\log n}{n^2}$

(18) $\displaystyle\sum_{n=1}^{\infty}\arctan n$

(19) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n!}$

(20) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}$

(21) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{3n-1}{5n+6}$

(22) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{2015^n}$

(23) $\displaystyle\sum_{n=2}^{\infty}\left(\frac{2n^3+1}{n^3-1}\right)^n$

(24) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n-1}{n}\right)^{n^2}$

(25) $\displaystyle\sum_{n=1}^{\infty}\frac{3^{n-1}}{2^n}$

(26) $\displaystyle\sum_{n=1}^{\infty}\frac{4^{1-n}}{3}$

(27) $\displaystyle\sum_{n=1}^{\infty}\frac{(-5)^n}{2^{3n}}$

Problem 5
Find the radius of convergence and interval of convergence of the following power series

(28) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2(-1)^n x^n}{4^n}$

(29) $\displaystyle\sum_{n=1}^{\infty}\frac{n!x^n}{n^2}$

(30) $\displaystyle\sum_{n=1}^{\infty}\frac{e^n x^n}{n!}$

(31) $\displaystyle\sum_{n=2}^{\infty}\frac{(x-1)^n}{n^2+1}$

(32) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n x^n$

Problem 6
Express the following functions as power series (Indicate radius of convergence and interval of convergence)

(33) $\dfrac{1}{10-x}$

(34) $\dfrac{x}{5+x^3}$

(35) $\dfrac{x^3}{(1-x)^2}$

(36) $\arctan(4x)$

Problem 7
Find the Taylor polynomial of degree 3 of the following functions

(37) $\sqrt[3]{x}$ centered at 1

(38) $\sin x$ centered at π

Problem 8
Find the general solution to the following ODEs

(39) $y' = xe^{-y}$

(40) $yy' + xy^2 = 4x$

(41) $3y' + y = 2e^{-x}$

(42) $y' - y\cos x = \dfrac{e^{\sin x}}{1+x^2}$

(43) $y'' - 2y' - 3y = e^{4x}$

(44) $y'' + y' = x - x^3$

Problem 9
Find the solution to the following initial value problems

(45) $y' = \dfrac{y^2 - 1}{xy} \quad y(1) = 1$

(46) $y' = 2\sqrt{y} \quad y(2) = 2$

(47) $y' = y + x \quad y(1) = e - 2$

(48) $y'' + 6y' + 9y = 0 \quad y(0) = 1, y'(0) = 2$

(49) $y'' + 4y = x \quad y(0) = 0, y'(0) = 1$

Problem *
Solve the following integrals

(50) $\displaystyle\int e^{x+e^x}\,\mathrm{d}x$

(51) $\displaystyle\int \sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$

(52) $\displaystyle\int x\sqrt{1-x}\,\mathrm{d}x$