# 05 – Matrix Inverses

**Definition: Elementary Matrix**

An **elementary matrix** a matrix obtained by performing a single elementary row operation on the identity $I$.

**1.** Determine if each of the following are elementary matrices.

**(a)** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

**(b)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**(c)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

**(d)** $\begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**(e)** $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 0 \\ \frac{1}{2} & 3 & 0 \end{bmatrix}$

**(f)** $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

**2.** Let $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ and let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Compute $EA$. What do you notice?

**Theorem**

Let $A$ be $m \times n$. If $\rho$ is an elementary row operation and $E = \rho(I)$ is the corresponding elementary matrix, then $\rho(A) = EA$. Moreover, $E$ is invertible with $E^{-1} = \rho^{-1}(I)$.

**Theorem**

Let $A$ be an $n \times n$ matrix. Then $A$ is invertible if and only if its RREF is $I_n$, and this happens if and only if $A$ is a product of elementary matrices. Further, when $A$ is invertible, any sequence of row operations that transforms $A$ to $I_n$ will also transform $I_n$ to $A^{-1}$.

**Theorem: Algorithm for finding $A^{-1}$**

If $A$ is $n \times n$, row reduce the augmented matrix $\left[A \mid I_n\right]$ to RREF.

- If the RREF of $\left[A \mid I_n\right]$ is $\left[I_n \mid B\right]$, then $A$ is invertible, and $B = A^{-1}$.
- If the RREF of $\left[A \mid I_n\right]$ is $\left[\text{“}\textit{not } I_n\text{”} \mid B\right]$, then $A$ is not invertible.

**3.** Find the inverse of $A$, if it exists.

**(a)** $\begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

**(b)** $\begin{bmatrix} 0 & 3 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

**(c)** $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$

**(d)** $\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 4 & -3 & 7 \end{bmatrix}$