Survey of Chapter 4

Linear algebra applies beyond $\mathbb{R}^n$.

$\boxed{\text{HO-07}}$ Def. of vector space

$\boxed{\text{HO-07}}$ Examples

Our definitions of lin. indep., span, basis, dimension, and lin. transformation apply to this more general coordinates.

$\boxed{\text{HO-07}}$ # 1

Fact

1. $\{1, t, t^2, \ldots, t^n\}$ is a basis for $\mathbb{P}_n$ so $\mathbb{P}_n$ has dimension $n+1$.

2. $\{1, t, t^2, \ldots\}$ is a basis for $\mathbb{P}$ — it is infinite dimensional

Coordinates

Theorem Let $\mathcal{B} = \{\bar{b}_1, \bar{b}_2, \ldots, \bar{b}_n\}$ be a basis for V. Then for each $\bar{v}$ in V, there is a unique set of scalars $c_1, c_2, \ldots, c_n$ such that

$$\bar{v} = c_1\bar{b}_1 + c_2\bar{b}_2 + \cdots + c_n\bar{b}_n .$$

* The vector $[\bar{v}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$ is called the <u>coordinate vector</u> of $\bar{v}$ relative to $\mathcal{B}$.

* coordinate vectors allow us to work with an n-dimensional vectorspace like it is $\mathbb{R}^n$. For example...

<u>Fact</u> Let $\bar{v}_1, \dots, \bar{v}_n$ be in V, and let $\mathcal{B}$ be a basis for V.

- $\bar{v}_1, \dots, \bar{v}_n$ are L.I. $\iff$ $[\bar{v}_1]_{\mathcal{B}}, \dots, [\bar{v}_n]_{\mathcal{B}}$ are L.I.

- if $\dim V = m$, then

$\bar{v}_1, \dots, \bar{v}_m$ are a basis for V $\iff$ $[\bar{v}_1]_{\mathcal{B}}, \dots, [\bar{v}_n]_{\mathcal{B}}$ are a basis for $\mathbb{R}^m$

$\boxed{\text{HO-07}}$ #2, 3

OPTIONAL

But what about linear transformations.

<u>Recall</u>: If $T: \mathbb{R}^n \to \mathbb{R}^m$ is linear, then the standard matrix for T is the $m \times n$ matrix

$$A = \left[T(\bar{e}_1) \cdots T(\bar{e}_n) \right]$$

and $T(\bar{v}) = A\bar{v}$ for all $\bar{v}$ in $\mathbb{R}^n$

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Theorem Let $T: V \to W$ be a linear trans. with V n-dimensional and W m-dimensional. Let $\mathcal{B} = \{\bar{b}_1, \dots, \bar{b}_n\}$ be a basis for V, and $\mathcal{B}'$ be a basis for W. The matrix representing T w.r.t. $\mathcal{B}$ and $\mathcal{B}'$ is the $m \times n$ matrix

$$A = \left[[T(\bar{b}_1)]_{\mathcal{B}'} \cdots [T(\bar{b}_n)]_{\mathcal{B}'} \right]$$

and $[T(\bar{v})]_{\mathcal{B}'} = A[\bar{v}]_{\mathcal{B}}$ for all $\bar{v}$ in V

* so again, every linear trans. can be viewed as a matrix trans.

$\boxed{\text{HO-07}}$ #4

The End