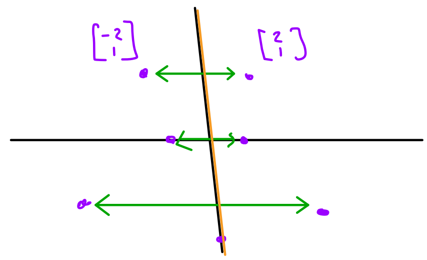
5.1 Eigenvectors + Eigenvalues

Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be reflection over the y-axis.



Q: what are some special subspaces associated to T?

* the y-axis: if $\bar{v}$ is on the y axis then $T(\bar{v}) = \bar{v}$.
* the x-axis: if $\bar{v}$ is on the x axis then $T(\bar{v}) = -\bar{v}$.

Def Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear trans. A scalar λ is called an eigenvalue of T if $T(\bar{x}) = \lambda \bar{x}$ has a nontrivial solution. Each nontrivial solution to $T(\bar{x}) = \lambda \bar{x}$ is called an eigenvector associated to λ. The collection of all solutions to $T(\bar{x}) = \lambda \bar{x}$ is called the eigenspace of T associated to λ, which we denote $E_\lambda(T)$.

* eigenvectors are nonzero but eigenvalues may be zero.
* eigenvectors are those that are just scaled by T.

Ex Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be refl. over the y-axis.

- $\lambda = 1$ is an eigenvalue b/c $T(\bar{x}) = \bar{x}$ has nontrivial sol.
  - $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ is an eigenvector assoc. to $\lambda = 1$, b/c $T\left(\begin{bmatrix} 0 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$
  - $E_1(T)$ is the collection of all vectors of the form $\begin{bmatrix} 0 \\ b \end{bmatrix}$, b in $\mathbb{R}$.

- $\lambda = -1$ is an eigenvalue
  - $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ is an eigenvector assoc. to $\lambda = -1$ b/c $T\left(\begin{bmatrix} 5 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$.
  - $E_{-1}(T)$ is the collection of all vectors of the form $\begin{bmatrix} a \\ 0 \end{bmatrix}$, a in $\mathbb{R}$.
- There are no other eigenvalues (or eigenvectors)
  - $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is not an eigenvector b/c $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \neq \lambda \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ for any λ.

Remember that lin. trans. can be represented by matrices. Let's translate our def. to matrices...

__Def__ Let A be $n \times n$. A scalar λ is an __eigenvalue__ of A if $A\bar{x} = \lambda \bar{x}$ has a nontrivial solution. Each nontrivial sol. to $A\bar{x} = \lambda\bar{x}$ is called an __eigenvector__ associated to λ; the collection of all solutions to $A\bar{x} = \lambda\bar{x}$ is the __eigenspace__.

__Ex__ Let $A = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$.

(a) Is $\bar{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ an eigenvector of A?

$A\bar{v} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ but $\begin{bmatrix} 5 \\ 2 \end{bmatrix} \neq \lambda \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ for any λ. Thus $\boxed{\text{NO}}$

(b) Is $\bar{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ an eigenvector of A?

$A\bar{w} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} = 5 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 5\bar{w}$. Thus, $\boxed{\text{Yes}}$ $\bar{w}$ is an eigenvector assoc. to the __eigenvalue__ $\lambda = 5$.

Prop Let A be $n \times n$ and λ be any scalar.

- λ is an eigenvalue of $A \iff (A-\lambda I)\bar{x} = \bar{0}$ has a nontrivial solution

- the eigenvectors associated λ are precisely the nontrivial solutions to $(A-\lambda I)\bar{x} = \bar{0}$

- If λ is an eigenvalue, then $E_\lambda(A) = \mathrm{Nul}(A-\lambda I)$. Thus eigenspaces are subspaces.

Pf

$$A\bar{v} = \lambda\bar{v} \iff A\bar{v} - \lambda\bar{v} = \bar{0} \iff A\bar{v} - \lambda I\bar{v} = \bar{0} \iff (A-\lambda I)\bar{v} = \bar{0} \quad \square$$

Ex Let $A = \begin{bmatrix} 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$.

(a) Show that $\lambda = 4$ is an eigenvalue and find a basis for $E_4(A)$.

$$A - 4I = \begin{bmatrix} 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

we now solve $(A-4I)\bar{x} = \bar{0}$

$$\left[\begin{array}{cccc|c} -1 & 0 & 2 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 2x_3$
$x_2 = 3x_3$
$s = x_3 = \text{free}$
$t = x_4 = \text{free}$

$$\bar{x} = \begin{bmatrix} 2s \\ 3s \\ s \\ t \end{bmatrix} = s\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad s, t \text{ in } \mathbb{R}$$

- Since $(A-4I)\bar{x} = \bar{0}$ has nontrivial sol., 4 is an eigenvalue of A

- A basis for $E_4(A)$ is $\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ so $\dim E_4(A) = 2$

- just for fun...

$$A \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 4 \\ 0 \end{bmatrix} = 4\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

(b) Show that 3 is not an eigenvalue of A.

$$A - 3I = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now solve $(A-3I)\bar{x} = \bar{0}$

$$\left[\begin{array}{cccc|c} 0 & 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

no free variables so only the trivial solution

$\bar{x} = \bar{0}$

- There is only the trivial sol, so 3 is <u>not</u> an eigenvalue of A.

5.2 The characteristic equation

... we have a geometric idea of eigenvectors and values.

... we know how to find eigenvalues once we know the eigenvalues

Q: How do we find the eigenvalues?

We know that...

λ is an eigenvalue of A $\iff$ $(A-\lambda I)\overline{x}=\overline{0}$ has nontrivial solutions

$\iff$ $(A-\lambda I)$ is not invertible

invertible matrix theorem — Sect. 2.3

$\iff$ $\det(A-\lambda I)=0$

Def Let A be $n \times n$. The <u>characteristic equation</u> of A is

$$\det(A-\lambda I)=0.$$

* $p(\lambda)=\det(A-\lambda I)$ is the <u>characteristic polynomial</u>.

* $p(\lambda)$ will have degree n.

<u>Ex</u> Let $A=\begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$. Find the char. poly of A.

$$p(\lambda)=\det(A-\lambda I)=\det\left(\begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}-\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)$$

$$=\det\begin{bmatrix} 2-\lambda & 7 \\ 7 & 2-\lambda \end{bmatrix}=(2-\lambda)^2-49=\lambda^2-4\lambda-45$$

Thus,

$$\boxed{p(\lambda) = \lambda^2 - 4\lambda - 45}$$

<u>Theorem</u> The eigenvalues of A are exactly the roots of the characteristic polynomial of A (i.e. the solutions to the char. equation).

<u>Ex</u> Find the eigenvalues of each matrix.

① $A = \begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$

char. poly is $p(\lambda) = \lambda^2 - 4\lambda - 45 = (\lambda - 9)(\lambda + 5)$

eigenvalues are $\boxed{\lambda = -5, 9}$

② $B = \begin{bmatrix} 3 & 4 & 11 \\ 0 & -5 & 7 \\ 0 & 0 & 3 \end{bmatrix}$.

char poly is $p(\lambda) = \det(B - \lambda I)$

$$= \det\left(\begin{bmatrix} 3 & 4 & 11 \\ 0 & -5 & 7 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right)$$

$$= \begin{vmatrix} 3-\lambda & 4 & 11 \\ 0 & -5-\lambda & 7 \\ 0 & 0 & 3-\lambda \end{vmatrix}$$

$$= (3-\lambda)(-5-\lambda)(3-\lambda)$$

eigenvalues are $\boxed{3, 3, -5}$ ← <u>multiplicity</u> of 3 is 2

Theorem If A is an upper or lower triangular $n \times n$ matrix, then the eigenvalues of A are precisely the entries on the main diagonal.

Similarity

Def Let A, B be $n \times n$. Then A is similar to B if there is an invertible matrix P such that

$$B = P^{-1} A P$$

* This is the same as $A = P B P^{-1} = (P^{-1})^{-1} B P$
so A similiar to $B \iff B$ similar to A.

Prop (similar matrices are very similar). Let A, B be similar $n \times n$ matrices. Then

1. $\det A = \det B$
2. A is invertible $\iff$ B is invertible
3. A and B have the same char. poly. Thus, they have the same eigenvalues with the same multiplicities.

pf Write $B = P^{-1} A P$ for some invertible matrix P.

1. $\det B = \det(P^{-1} A P) = \det P^{-1} \cdot \det A \cdot \det P$

$$= \frac{\det A \cdot \det P}{\det P}$$

$$= \det A.$$

(2)

$$
\begin{aligned}
P_B(\lambda) &= \det(B - \lambda I) \\
&= \det(P^{-1} A P - \lambda I) \\
&= \det(P^{-1} A P - \lambda P^{-1} P) \\
&= \det\left[P^{-1}(AP - \lambda P)\right] \\
&= \det\left[P^{-1}(A - \lambda I) P\right] \\
&= \det(A - \lambda I) \\
&= P_A(\lambda).
\end{aligned}
$$

! Row operations usually do change eigen values, but similarity does not.

5.3 Diagonalization

Thinking about powers of a matrix...

Q: Let $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ ← diagonal matrix. what is D^7?

ERROR First time through

$$D^7 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\cdots$$

$$= \begin{bmatrix} 2^7 & 0 \\ 0 & 1^7 \end{bmatrix} = \begin{bmatrix} 128 & 0 \\ 0 & 1 \end{bmatrix}$$

* so D^k is easy to compute if D is diagonal.

Q: Let $A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$. what is A^7?

$$A^7 = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}\cdots$$

However, it is true that A is similar to a diagonal matrix:

$$A = PDP^{-1} \quad \text{where} \quad D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

Notice that

$$A^7 = (PDP^{-1})^7$$

$$= PD\cancel{P^{-1}P}D\cancel{P^{-1}P}DP\cdots$$

$$= PD^7P^{-1}$$

246   -128

1   -1

Thus,

$$A^7 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 128 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 255 & -127 \\ 254 & -126 \end{bmatrix}$$

not so bad

* So, A^k is pretty easy to compute if A is similar to a diagonal matrix

<u>Prop</u> If $A = PDP^{-1}$, then $A^k = PD^kP^{-1}$.

<u>Def</u> A matrix is <u>diagonalizable</u> if it is similar to a diagonal matrix.

Q: It seems valuable to know if a matrix is diagonalizable, but how can we determine this?

Suppose $A = PDP^{-1}$ where $D = \begin{bmatrix} d_1 & & 0 \\ & \ddots & \\ 0 & & d_n \end{bmatrix}$. Then

$$AP = PD$$

Let's compare columns of LHS and RHS.

Write $P = [\bar{v}_1 \cdots \bar{v}_n]$.

$$AP = \left[A\bar{v}_1 \;\; A\bar{v}_2 \cdots A\bar{v}_n \right]$$

$$PD = \left[P\begin{bmatrix} d_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \;\; P\begin{bmatrix} 0 \\ d_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdots P\begin{bmatrix} 0 \\ \vdots \\ 0 \\ d_n \end{bmatrix} \right]$$

$$= \left[d_1\bar{v}_1 \;\; d_2\bar{v}_2 \cdots d_n\bar{v}_n \right]$$

Thus $A\bar{v}_1 = d_1\bar{v}_1$, $A\bar{v}_2 = d_2\bar{v}_2$, ..., $A\bar{v}_n = d_n\bar{v}_n$

Whoa... the entries of D are eigenvalues of A and the columns of P are assoc. eigenvectors!

Diagonalization Theorem Let A be $n \times n$.

Then A is diagonalizable if and only if

A has n linearly independent eigenvectors.

Further, if $\bar{v}_1, \dots, \bar{v}_n$ are lin. ind. eigenvectors corresponding to the eigenvalues $\lambda_1, \dots, \lambda_n$, then $A = PDP^{-1}$ where

$$P = \begin{bmatrix} \bar{v}_1 & \bar{v}_2 & \cdots & \bar{v}_n \end{bmatrix}$$

$$D = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

Ex Diagonalize the following if possible.

(a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

mult. 2

(a) A is lower Δ, so eigenvalues are $\boxed{\lambda = 1, 2}$

$\circledast$ $\underline{\lambda = 1}$ $E_1(A) = \text{Nul}(A - I) = \text{Nul}\left(\begin{bmatrix} 0 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \begin{array}{cccc} x_1 & \overset{s}{\underset{\|}{}} x_2 & \overset{t}{\underset{\|}{}} x_3 & \\ \hline \end{array}\left[\begin{array}{ccc|c} 1 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \bar{x} = \begin{bmatrix} -\frac{1}{3}s \\ s \\ t \end{bmatrix} = s\begin{bmatrix} -1/3 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

A Basis for $E_1(A)$ is $\left\{ \begin{bmatrix} -1/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

thus we have 2 L.I. eigenvectors so far

$\lambda = 2$

$$E_2(A) = \text{Nul}(A - 2I) = \text{Nul}\left(\begin{bmatrix} -1 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}\right)$$

$$\left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \sim \begin{array}{c} \begin{array}{ccc} x_1 & \overset{s}{\overset{\shortparallel}{x_2}} & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \qquad \bar{x} = \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

A Basis for $E_2(A)$ is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

Let's try to combine our bases

$$\left\{ \begin{bmatrix} -1/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$ is easily seen to be LI.

This combining process always yields a L.I. set.

A is 3×3, and we found 3 L.I. eigenvectors. Thus,

A is diagonalizable as $A = PDP^{-1}$ with

$$P = \begin{bmatrix} -1/3 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(b) Eigenvalues of B are again $\lambda = 1, 2$

(*) $\underline{\lambda = 1}$ $E_1(B) = \text{Nul}(B - I) = \text{Nul}\left(\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \sim \begin{array}{c} \begin{array}{cccc} x_1 & \overset{s}{\overset{\shortparallel}{x_2}} & x_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \qquad \bar{x} = \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

A basis for $E_1(B)$ is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(*) $\underline{\lambda = 2}$ $E_2(B) = \text{Nul}(B - 2I) = \text{Nul}\left(\begin{bmatrix} -1 & 0 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$

$$\left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \qquad \bar{x} = \begin{bmatrix} 0 \\ 0 \\ s \end{bmatrix} = s\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

A basis for $E_2(B)$ is $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(*) There are at most 2 L.I. eigenvectors — one from $E_1(B)$ and one from $E_2(B)$.

Thus, B does not have 3 L.I. eigenvectors, so B is <u>NOT</u> Diagonalizable.

Note that each eigenspace contributes at least one eigenvector so...

<u>Theorem</u> Let A be $n \times n$. If A has n different eigenvalues, then A is diagonalizable.

* If A does not have n different eigenvalues, A may or may not be diagonalizable.

Ex Explain, quickly, why each of the following are diagonalizable.

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 6 & 11 & \pi & 0 \\ 1 & 2 & 3 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

- A is 4×4 and has 4 distinct eigenvalues: $\lambda = 5, 0, \pi, -1$
- char poly of B is $\begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda) - 6$

$$= \lambda^2 - 5\lambda - 2$$

quad. formula says roots are $\dfrac{-5 \pm \sqrt{33}}{2}$, so

B is 2×2 and has 2 distinct eigenvalues: $\lambda = \dfrac{-5 \pm \sqrt{33}}{2}$