Applied Linear Algebra — Outline for Exam 1

Sections 1.1–1.7

Main ideas

A. Solving linear systems, vector equations, and matrix equations

B. Solution sets: writing in parametric vector form and interpreting geometrically

C. Application(s): network flow

D. Key terms

- row echelon form (REF) and reduced row echelon form (RREF)
- consistent/inconsistent equations
- free variables
- homogeneous linear system
- linear combination
- the set spanned by a collection of vectors
- linear independence/dependence

Skills you should have

1. Be able to row reduce a matrix to RREF.

2. Be able to solve linear systems.

- *Usual process:* (1) write as augmented matrix, (2) row reduce to REF or RREF, (3) write the solution set (using free variables if necessary).
- Be able to determine if the system is consistent/inconsistent and if there are infinitely many solutions.
- Be able to write the solution set in parametric vector form.
- Be able to interpret "small" solution sets geometrically: point, line, or plane.

3. Be able to multiply a matrix by a vector.

4. Be able to solve linear vector and matrix equations; that is, equations of the form $x_1\mathbf{v}_1+x_2\mathbf{v}_2+\cdots+x_n\mathbf{v}_n = \mathbf{b}$ and $A\mathbf{x} = \mathbf{b}$, for A an $m \times n$ matrix.

- *Usual process:* convert to an augmented matrix and then solve as a linear system.
- Sometimes these can be solved by simple guessing and checking.

5. Be able to determine if a vector $\mathbf{b}$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$.

- Note: this problem is the same as determining if $\mathbf{b}$ is in Span$\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$; it is also the same as determining if $A\mathbf{x} = \mathbf{b}$ is consistent, where A is the matrix with columns $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$.
- *Usual process:* determine if $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_k\mathbf{v}_k = \mathbf{b}$ is consistent by converting to an augmented matrix and solving as a linear system.
- Be able to determine if *every* $\mathbf{b} \in \mathbb{R}^m$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$.

  - Usual process: (1) make the "coefficient" matrix whose columns are $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$, (2) row reduce to REF, (3) if there is a pivot in every *row*, then YES, every $\mathbf{b} \in \mathbb{R}^m$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$; if there is NOT a pivot in every *row*, then NO.

6. Be able to determine if $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent or linearly dependent.

- *Usual process:* $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly *independent* if and only if $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_k\mathbf{v}_k = \mathbf{0}$ has only one solution (i.e. if there are no free variables).
- There are also some theorems that can sometimes (but not always) help.
  - Assume $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are in $\mathbb{R}^m$. If $k \geq m$, then the vectors must be linearly dependent.
  - If one of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ is the zero vector, then the vectors must be linearly dependent.
  - Two vectors $\mathbf{v}_1, \mathbf{v}_2$ are linearly dependent if and only if one is a scalar multiple of the other.

7. Be able to solve network flow problems.

- The main principle is that "flow in = flow out" at every intersection. Also, the flow into the entire network must equal the flow out of the entire network.

How to study

I. Review core topics

II. Work *lots* of problems all of the way through—focus on WeBWorK problems, Homework problems, and Handout problems.

- WeBWork #1–5, Homework #1–3, Handout #1–4.

III. Practice doing several problems in a short amount of time (by timing yourself)

IV. Come talk with me if you have any questions