Applied Linear Algebra — Outline for the Final Exam

Anything that is crossed out will NOT be on the final exam!

Main ideas

OLD

A. Solving linear systems, vector equations, matrix equations

B. Solution sets: writing in parametric vector form and interpreting geometrically

C. Linear combinations, linear independence, span, basis, dimension

D. Linear transformations

E. Matrix operations and determinants

F. ~~Applications: network flow and volume of parallelograms~~

NEW

G. Eigenvalues, eigenvectors, eigenspaces, and the characteristic polynomial

H. Similarity and diagonalization

I. Inner product (i.e. dot product), length, distance, and orthogonality

J. Orthogonal projection

K. Applications: PageRank and least squares solutions

Skills you should have

1. Be able to solve linear systems.

- Be able to determine if the system is consistent/inconsistent and if there are infinitely many solutions.
- Be able to write the solution set in parametric vector form.
- Be able to interpret "small" solution sets geometrically: point, line, or plane.

2. Be able to determine if a vector $\mathbf{b}$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$.

- *Usual process:* determine if $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_k\mathbf{v}_k = \mathbf{b}$ is consistent by converting to an augmented matrix and solving as a linear system.

3. Be able to determine if $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent or linearly dependent.

- *Usual process:* $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly *independent* if and only if $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_k\mathbf{v}_k = \mathbf{0}$ has only the trivial solution (i.e. if there are no free variables).

4. Be able to determine if a set of vectors is a basis for a subspace

5. Be able to find a basis for subspaces (and their dimension) in the following situations:

- Be able to find a basis for the null space of a matrix
- Be able to find a basis for the column space of a matrix

6. Be able investigate and work with linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$
   - Be able to find the standard matrix for a linear transformation
   - Be able to determine if a given vector is in the range of linear transformation
   - Be able to determine if a linear transformation is one-to-one or onto (or both)
7. Be able to perform matrix operations
   - This includes addition, subtraction, multiplication, and the transpose
   - Be able to use row reduction to find the inverse of a matrix, if it exists
   - Know how to use A^{-1} to solve $A\mathbf{x} = \mathbf{b}$
8. Be able to compute determinants (using cofactor expansion or row-reduction to triangular form)
9. Be able to apply properties of the determinant
   - Two key properties are $\det(AB) = (\det A)(\det B)$ and $\det(A^{-1}) = (\det A)^{-1} = \frac{1}{\det A}$ (when $\det A \neq 0$)
   - Know that A is invertible if and only if $\det A \neq 0$
10. Be able to find the eigenvalues and corresponding eigenspaces for a matrix A.
11. Be able to determine if a matrix A is diagonalizable, and if it is, be able to find a diagonal matrix D and invertible matrix P such that $A = PDP^{-1}$.
12. Be able to apply use the PageRank process to rank webpages
    - *Usual process:* (1) find the transition matrix, (2) find an eigenvector $\mathbf{v}$ associated to $\lambda = 1$ (I'll probably do this for you), (3) divide $\mathbf{v}$ by the sum of its entries to obtain the steady-state vector, (4) use the probabilities in the steady-state vector to rank the webpages.
13. Be able to work with the inner product (i.e. dot product).
    - Be able to compute the inner product, length, and distance
    - Know that two vectors are orthogonal if and only if their inner product is 0
    - Know the definition of $W^\perp$ for W a subspace of $\mathbb{R}^n$
14. Be able to find the orthogonal projection of a vector onto a subspace
    - I will only ask you to find the projection onto a subspace of dimension 1 or 2.
15. Be able to find a least squares solution to a linear system
    - *Usual process:* instead of solving $A\mathbf{x} = \mathbf{b}$, you solve $A^T A\mathbf{x} = A^T\mathbf{b}$
    - Be able to apply this to fitting a linear function or quadratic function to data points

How to study

I. Review core topics

II. Work *lots* of problems all of the way through—focus on WeBWorK problems, Homework problems, and Handout problems (but *not* Handout 07).

III. Practice doing several problems in a short amount of time (by timing yourself)

IV. Come talk with me if you have any questions