Math 100—Homework 02

Due: Friday February 15

Name ______________________

Directions: please print this page, and put your solutions in the space provided.
If you need extra space, you can attach another sheet of paper.

1. Let $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}, \mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} -4 \\ -2 \\ 3 \end{bmatrix}$. Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$. If it is, find values for x_1, x_2, x_3 such that $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{b}$. *Show all work.*

2. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$. Determine if $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$. *Show all work and explain.*

3. Consider the matrix $A = \begin{bmatrix} 0 & 1 & 1 & -5 \\ 1 & 3 & -2 & 2 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$. Is it true that that $A\mathbf{x} = \mathbf{b}$ has a solution for *every* possible choice of $\mathbf{b}$ in $\mathbb{R}^4$? *Show all work and explain.*

4. Suppose your friends tells you "I found four vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$ in $\mathbb{R}^6$, and I'm pretty sure that every vector in $\mathbb{R}^6$ can be written as a linear combination of $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$." Even though your friend hasn't told you which four vectors they are thinking of, you know that they are wrong. Use one of our Theorems from class to carefully explain why your friend must be wrong.