Math 100—Homework 04

Due: Friday March 15

Name _____

Directions: please print this page, and put your solutions in the space provided.
If you need extra space, you can attach another sheet of paper.

1. Consider the following linear transformations.

- $R : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that rotates points *clockwise* by an angle of $\frac{\pi}{3}$.
- $S : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that projects points onto the y-axis, so $S(x_1, x_2) = (0, x_2)$.
- $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the composition $S \circ R$, so $T(\mathbf{x}) = S(R(\mathbf{x}))$.

(a) Find the standard matrix for R, and call it A. *Make sure to show all work.*

$A =$

(b) Find the standard matrix for S, and call it B. *Make sure to show all work.*

$B =$

(c) Find the standard matrix for T, and call it C. *Make sure to show all work.*

$C =$

(d) Compute the matrix products AB and BA. *Hint: one of your answers for this part should turn out to be the same as one of the matrices you found above.*

$AB =$

$BA =$

2. Suppose that $T : \mathbb{R}^7 \to \mathbb{R}^5$ is a linear transformation.

(a) If A denotes the standard matrix for T, what are the dimensions of A, i.e. how many rows and columns? *Briefly explain.*

(b) Carefully explain, using pivots, why T cannot possibly be one-to-one.

3. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & -3 \end{bmatrix}$

(a) Compute A^{-1}.

(b) Use A^{-1} to solve the equation $Ax = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$. (Please use A^{-1}, not row reduction.)