Math 108—Outline for Final Exam

Problems: what we did in class, plus what I did on the last Wednesday (up through 7.27)

Definitions

One thing that I hope you all take away from this course is a fluency in the language of logic and set theory. To that end, you are expected to be able to recite the definitions for the following terms.

- the **converse** and **contrapositive** of an implication $P \implies Q$
- what it means to say that a set A is a **subset** of a set B
- the **union**, **intersection**, and **difference** of two sets
- the **power set** of a set S (symbolically $\mathcal{P}(S)$)
- the **Cartesian product** of sets X and Y (symbolically $X \times Y$)
- a **relation** on a set X
- the **reflexive**, **symmetric**, and **transitive** properties for a relation $\sim$ on a set X
- the **set of relatives of** x when $\sim$ is a relation on A (symbolically $[x]$)
- an **equivalence relation** on a set X
- a **function** and its **domain**, **codomain**, and **range**
- **one-to-one** (injection), **onto** (surjection), and **bijection**
- the **inverse** of a function

Problems to Practice Redo the exercises from the notes!

From Exam 1

1. Translating to and from symbolic logic
2. Proving two propositional forms are equivalent with a truth table
3. Using set-builder notation
4. "True or False" problems (with explanations)
   - **(a)** Determining truth values of propositions (especially involving quantifiers)
   - **(b)** Determining if an element is contained in a given set
   - **(c)** Determining if a set is contained in a given set

From Exam 2

5. Being able to provide examples of elements and/or sets meeting certain set-theoretic criteria
6. Working with Cartesian products
7. Working with unions and intersections over families of sets
8. Checking if a relation is reflexive, symmetric, and transitive
9. Working with equivalence relations—for example, determining $[x]$ (the equivalence class of an element)
10. Checking if a collection of subsets of A is a partition of A
11. "Example or no such example exists" problems

New Material

12. Determining if a relation is a function
13. Determining if a function is one-to-one or onto (or both)
14. Determining if a function has an inverse

Proofs Redo the theorems from the notes!

From Exam 1

1. Practice choosing which proof technique to use (e.g. contraposition or cases, like in Theorem 2.80)
2. Practice "prove or disprove" problems (these are easy to make up on your own)
3. Practice how to prove that a set is contained in another set and how to prove that two sets are equal

From Exam 2

4. Practice proving theorems with Induction
5. Practice proving (or disproving) the reflexive, symmetric, and transitive properties for a relation
6. Practice proving properties about equivalence relations
7. Re-practice the "basics," for example: how to prove implications, how to prove that a set is contained in another set, or how to prove that two sets are equal

New Material

8. Practice how to prove or disprove that a function is an one-to-one or onto (or both)
9. Be ready to prove the following theorems from the notes: Theorem 7.20, 7.21, 7.23(a), 7.23(d)