


MATH 108—WRITING ASSIGNMENT 12

Due: Friday December 8—4:00PM

Getting Started

1. Get the template for this assignment. Here's how to do it:
 - Go to <https://www.sharelatex.com>, and **make sure you are logged in**.
 - In a new window, go here:
<https://www.sharelatex.com/read/dbkvfwbsyhcm>
 - Click on the menu icon  and select “Copy Project”
 - When ask for a name, choose something like “Math 108 - WA 12” and click “Copy”
 - When this completes you will be back in your own workspace (instead of mine).
 - After solving the problems (possibly with your peers), type them up using this template.
 - Email me (or print and turn in) your final draft.
2. Let me know if you have any questions!

Problems are below.

1. Let $(0, 1]$ be the interval defined by $(0, 1] = \{x \in \mathbb{R} \mid 0 < x \leq 1\}$. Prove that $f : \mathbb{R} \rightarrow (0, 1]$ defined by $f(x) = \frac{1}{1+x^2}$ is a surjection, i.e onto, without referring to a graph.
Hint: begin with “Let $y \in (0, 1]$. We will prove that there exists an $x \in \mathbb{R}$ such that $f(x) = y$. Next, use the definition of f to write what $f(x) = y$ means, and show that you can solve for y . Make sure that you clearly highlight when you are using that $0 < y \leq 1$.
2. Prove that $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(m, n) = 2^{m-1}(2n - 1)$ is an injection, i.e. one-to-one. (In fact, f is also onto \mathbb{N} , but you do not need to prove this.)
Hint: begin with “Let $(m_1, n_1), (m_2, n_2) \in \mathbb{N} \times \mathbb{N}$, and assume $f(m_1, n_1) = f(m_2, n_2)$.” Next, use the definition of f to write what $f(m_1, n_1) = f(m_2, n_2)$ means, and then move things around. If you can force one side of the equation to be an odd integer, then this means that the other side must be odd too...