Math 110A—Outline for Exam 2

Sections covered: focus on Sections 3.1–5.1 (starting at Theorem 3.21 and ending at Theorem 5.8)

Overview of Topics

A. More about subgroups and subgroup lattices

B. Isomorphism

C. Families of groups: cyclic groups, dihedral groups, symmetric and alternating groups

D. Cosets of a subgroup

Skills you should have

1. Be able to prove if a subset of a group is subgroup or not.

- We did this before, but this time we tackled more abstract contexts, e.g. proving $Z(G)$ is a subgroup or that the interesection of subgroups is again a subgroup.

2. Be able to compute subgroups of a group and create the subgroup lattice.

- I would only ask you to construct the subgroup lattice for cyclic groups with few divisors (e.g. $\mathbb{Z}_{18}$) or noncyclic groups order at most 8.

3. Be able to work with isomorphisms

- Be able to prove if a given function is an isomorphism of groups or not.
- Be able to determine if familiar groups and/or possibly new groups defined via a group table are isomorphic or not.
- Know and be able to prove basic properties about isomorphisms.

4. Be able to work with cyclic groups.

- Know our various classes of cyclic groups: R_n, $\mathbb{Z}_n$, and U_n.
- Know how to find all generators of a cyclic group and be able to compute orders of elements.
- Be able to apply our core theorems about cyclic groups, e.g. subgroups of a cyclic group are cyclic.

5. Be able to work with dihedral groups.

- Just focus on the those of small order: D_3, D_4, D_5, D_6.

6. Be able to work with symmetric and alternating groups.

- Know the definitions of S_n and A_n and the order of each group.
- Know how to work with cycle notation and with permutation diagrams.
- Know the definitions of transposition, even permutation, and odd permutation.
- Know how to simplify a product of cycles to a product of disjoint cycles.
- Know how to expand a product of cycles to a product of transpositions.
- Be able to compute the orders of permutations when given cycle notation.
  - Simplify to a product of disjoint cycles (if needed), then use Theorem 4.80.

7. Be able to compute cosets of a subgroup of a group.

- Practice problems like 5.2–5.7
- Be able to prove basic theorems about cosets, e.g. Theorem 5.8 or 5.12, but you will not need to know or use Theorem 5.9 or later. You will just need to work with the definition of a coset.

8. As before, be able to give examples of groups with or without certain properties.

- You will want to be reasonably familiar with the groups we've worked with a bit: R_n $(n \geq 3)$, $\mathbb{Z}_n$ $(n \geq 3)$, U_n $(n \geq 3)$, D_n $(n = 3, 4)$, S_n $(n = 3, 4, 5)$, A_n $(n = 3, 4)$, V_4, L_n $(n = 2, 3)$, Q_8, $\mathbb{Z}$, $\mathbb{R}$, $(\mathbb{R} \setminus \{0\}, \cdot)$.

How to study

1. Review core topics—make sure to have a working understanding of all definitions and theorems.
2. Review the many examples of groups we have, and think about their properties, e.g. which are abelian, which are cyclic, what are the orders of them, which groups are isomorphic, what are some elements they have, what are some subgroups they have.
3. Work problems similar to those from our notes. (You can also look in other books for problems to try.)
4. Practice proofs similar to those from our notes. (You can also look in other books for proofs to try.)
5. Come talk with me if you have any questions!