MATH 110A—OUTLINE FOR THE FINAL EXAM

Sections covered: everything up to 7.2, except for 4.2—please focus on the new material: 5.1–7.2

Overview of Topics

  A. Groups and subgroups

  B. Generating sets and minimal generating sets

  C. Group tables and Cayley diagrams

  D. Subgroup lattices

  E. Examples (and nonexamples) of groups

  F. Homomorphism and Isomorphism

  G. Families of groups: cyclic groups, dihedral groups, symmetric and alternating groups

  H. Cosets of a subgroup and normal subgroups

  I. Products and Quotients

  J. Image and kernel of a homomorphism and the First Isomorphism Theorem

Skills you should have

  1. Be able to determine (and prove) if a subset of a group is subgroup or not

  2. Be able to construct and work with group tables and Cayley diagrams

  3. Be able to work with cyclic groups.

  - Know our various classes of cyclic groups: R_n, $\mathbb{Z}_n$, and U_n.
  - Be able to apply our core theorems about cyclic groups, e.g. subgroups of a cyclic group are cyclic and $g^k = e \implies |g|$ divides k.

  4. Be able to work with dihedral groups.

  - Just focus on the those of small order: D_3, D_4, D_5, D_6
  - The key relations for D_n are: $r^n = e$, $s^2 = e$, and $srs = r^{-1}$

  5. Be able to work with symmetric and alternating groups.

  - Know the relevant definitions: cycle, cycle notation, even/odd permutation, transposition, A_n
  - Know how to simplify a product of cycles to a product of disjoint cycles
  - Know how to expand a product of cycles to a product of transpositions
  - Be able to find the order of a permutation

  6. Be able to work with cosets of a subgroup

  - Be able to compute the cosets of a given subgroup in a given group
  - Be able to determine if a subgroup is normal or not
  - Know and be able to prove properties about cosets

  7. Be able to work with direct products

  - Know to compute orders of elements and determine if a product of groups is cyclic or abelian
  - Know and be able to prove properties about direct products

  8. Be able to work with quotient groups

  - Know to compute orders of elements and determine if a quotient is cyclic or abelian
  - Know and be able to prove properties about quotients

  9. Be able to work with homomorphisms

  - Be able to determine (and prove) if a given function is a homomorphism (or an isomorphism)

- Be able to determine the kernel and image of a homomorphism
- Be able to apply the First Isomorphism Theorem
- Know and be able to prove properties about homomorphisms

10. As before, be able to give examples of groups with or without certain properties.

   - Be reasonably familiar with the groups we've worked with: R_n $(n \geq 3)$, $\mathbb{Z}_n$ $(n \geq 3)$, U_n $(n \geq 3)$, D_n $(n = 3, 4)$, S_n $(n = 3, 4, 5)$, A_n $(n = 3, 4)$, V_4, L_n $(n = 2, 3)$, Q_8, $\mathbb{Z}$, $\mathbb{R}$, $(\mathbb{R} \setminus \{0\}, \cdot)$.
   - Be able to make new groups from old ones using products. For example, $\mathbb{Z}_4 \times \mathbb{Z}_6$ and $D_4 \times S_5$

11. Be able to prove or disprove statements abstract groups.

   - A good starting point is to make sure you can reprove the theorems that we have covered.
   - You will have approximately 4 proofs to complete on the final exam. One of them is the theorem below. Please be prepared to prove it. *You can discuss it with me or anyone in our class.*

   Theorem. Let $\phi : G \to H$ be a homomorphism between groups, and let $K = \ker(\phi)$. Then $\phi(a) = \phi(b)$ if and only if $aK = bK$.
   Hint: make use of our theorems about cosets—Theorem 5.14(b) may be helpful.

How to study

1. Review core topics—make sure to have a working understanding of all definitions and theorems.

2. Review the many examples of groups we have, and think about their properties, e.g. which are abelian, which are cyclic, what are the orders of them, which groups are isomorphic, what are some elements they have, what are some subgroups they have.

3. Work problems similar to those from our notes. (You can also look in other books for problems to try.)

4. Practice proofs similar to those from our notes. (You can also look in other books for proofs to try.)

5. Come talk with me if you have any questions!