# Math 110B—Outline for Exam 1

*Sections covered: all of Chapters 2, 3, 4.*

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**Overview of Topics**

**A.** The complex numbers $\mathbb{C}$, the quaternion numbers $\mathbb{H}$, and $\mathbb{Z}_n$

**B.** Fields: definition, basic properties and examples

**C.** Subfields and extension fields

**D.** Generating subfields (and adjoining elements to fields)

**E.** Subfield lattices

**F.** Radical extensions and solvability by radicals for polynomials

**Skills you should have**

**1.** Be able to work with and prove results about the complex numbers $\mathbb{C}$
- Be able to move back and forth between the $a + bi$ and the $r\cos\theta + i\sin\theta$ forms
- De Moivre's formula is very important, but it only applies for positive, integer powers
- Know properties of and be able to work with $\zeta_n$
- Be able to use Theorems 3.24 and 3.26 to find all $n^{\text{th}}$ roots of a complex number $b$. Remember that finding all $n^{\text{th}}$ roots of $b$ is the same as finding all roots of the polynomial $x^n - b$

**2.** Be able to work with and prove results about the quaternion numbers $\mathbb{H}$

**3.** Be able to work with and prove results about $\mathbb{Z}_n$ for various $n$

**4.** Be able to determine if a structure is a field/subfield or not
- Be able to do this given a table or an algebraic representation of the structure
- Work problems similar to Problems 3.44, 3.47, and 3.57 (but work other types of problems too)
- Be able to organize a collection of subfields into a lattice (like in Problems 3.73, 3.74)

**5.** Be able to prove basic properties about fields (similar to Theorem 3.50)

**6.** Be able to work with and prove results about subfields generated by adjoining elements.
- Try to work problems similar (by changing some numbers) to Problems 3.64–3.69. Theorem 3.68 can be useful
- Be able to prove Theorem 3.68 too.

**7.** Be able to show an extension field is a radical extention

**8.** Be able to show a polynomial is solvable by radicals
- Be able to do this in a concrete setting (Problem 4.14 is a nice, hard one to review)
- Be able to work in a more general setting (like Theorems 4.11, 4.12)

**Rules for the exam**

**1.** You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem 3.68, then you should say so.

**2.** Unless you prove them, you cannot use any results from the course notes that we have not yet covered.

**3.** You are **NOT** allowed to consult external sources when working on the exam. This includes people inside or outside of the class, other textbooks, and online resources. If in doubt, ask me.

**4.** You are **NOT** allowed to discuss the problems with anyone other than yourself and me (Josh).

**5.** You are **NOT** allowed to let someone else copy your work.

**How to study**

1. Review core topics—make sure to have a working understanding of all definitions and theorems.
2. Work problems similar to those from our notes. (You can also look in other books for problems to try.)
3. Practice proofs similar to those from our notes. (You can also look in other books for proofs to try.)
4. Be prepared for more open-ended problems like "true/false" or "prove/disprove" problems. Try making up ones to practice—you can try to stump your classmates.
5. Come talk with me if you have any questions!