Math 110B—Outline for Exam 2

Sections covered: all of Chapter 5; Chapter 6 up through Problem 6.32 .

Overview of Topics

A. Rings: definition, basic properties, and examples

B. Special types of rings: commutative rings, rings with unity, division rings, integral domains, PIDs

C. Units, zero divisors, irreducible elements, and reducible elemnts

D. Polynomial rings and matrix rings

E. Ideals and quotient rings

F. Ring homomorphisms and the First Isomorphism Theorem

G. Algebraic elements and minimal polynomials

H. Irreducibility tests

I. Describing the elements of $F(\alpha)$

Skills you should have

1. Know examples and non-examples of rings with special properties commutative rings, rings with unity, division rings, integral domains, PIDs, fields
   - Also be able to prove that the example has the given property or not
2. Be able to work with and prove results about rings, subrings, and ideals
   - Be able to determine if a structure is a ring/subring/ideal or not
   - Be able to organize a collection of ideals into a lattice (like in Problems 5.100, 5.104)
   - Theorems 5.91 and 5.99 are often helpful
3. Be able to determine if elements of a ring are units, zero divisors, or neither
4. Be able to work with and prove results about polynomial rings over fields
   - The main tools are the division algorithm (5.43), the existence of a greatest common divisor (5.54), and the fact that all ideals are principal (5.95)
5. Be able to determine if a polynomial is irreducible or not
   - If the degree is ≤ 3, you can use 5.63 and 5.64
   - For large degree polynomials try factoring or try using the EIC (6.25)—remember you might have to manipulate the polynomial before the EIC applies (see 6.28 and 6.30)
6. Be able to work with and prove results about quotient rings
   - The main tools are 5.74 and 5.76
   - See 5.78(4,5,6), 5.86, and 6.18
7. Be able to work with and prove results about homomorphisms, kernels, and images
   - Be able to determine if a function is a homomorphism or not
   - Be able to find the kernel and the image
   - Be able to apply the First Isomorphism Theorem (5.119)
8. Be able to find the minimal polynomial of an algebraic element α use it to describe $F(\alpha)$
   - Remember that if α is a root of any polynomial $p(x)$, then the minimal polynomial must be a factor of $p(x)$ (6.6)
   - To describe $F(\alpha)$ you can use 6.19
   - Be able to use the minimal polynomial to simplify expressions in $F(\alpha)$ (see 6.23)
   - Also be able to describe $F(\alpha, \beta)$ (see 6.24)

Rules for the exam

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Theorem 3.68, then you should say so.
2. Unless you prove them, you cannot use any results from the course notes that we have not yet covered.
3. You are **NOT** allowed to consult external sources when working on the exam. This includes people inside or outside of the class, other textbooks, and online resources. If in doubt, ask me.
4. You are **NOT** allowed to discuss the problems with anyone other than yourself and me (Josh).
5. You are **NOT** allowed to let someone else copy your work.

How to study

1. Review core topics—make sure to have a working understanding of all definitions and theorems.
2. Work problems similar to those from our notes. (You can also look in other books for problems to try.)
3. Practice proofs similar to those from our notes. (You can also look in other books for proofs to try.)
4. Be prepared for more open-ended problems like "true/false" or "prove/disprove" problems. Try making up ones to practice—you can try to stump your classmates.
5. Come talk with me if you have any questions!