Linear Algebra
MATH 224W – Spring 2016

Week 12: Rank, nullity, and linear transformations

Writing Assignment #11 **due Friday, Apr. 22**

§4.9 #45

AP #1 Let D be an $n \times n$ matrix, and define a function $L : M_{n\times n} \to M_{n\times n}$ by $L(A) = DA - AD$. Prove that L is a linear transformation.

AP #2 Let V and W be vector spaces, and let $T : V \to W$ be a *one-to-one* linear transformation. Prove that if $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a linearly independent set of vectors in V, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\}$ is a linearly independent set of vectors in W.
Hint: you want to show $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\}$ is linearly independent, so your proof should begin with "assume that $a_1T(\mathbf{v}_1) + \cdots + a_kT(\mathbf{v}_k) = \mathbf{0}_W$ for some $a_1, \ldots, a_k \in \mathbb{R}$." Now you need to show that this implies that $a_1 = \cdots = a_k = 0$. It is helpful to note that $\mathbf{0}_W = T(\mathbf{0}_V)$, but if you use this, make sure to quote the relevant theorem.

AP #3 Let V and W be vector spaces, and let $T : V \to W$ be an *onto* isomorphism. Prove that if $\operatorname{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_k\} = V$, then $\operatorname{span}\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\} = W$.
Hint: you want to show $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\}$ spans W, so your proof should begin with "let $\mathbf{w} \in W$; we will show that $a_1T(\mathbf{v}_1) + \cdots + a_kT(\mathbf{v}_k) = \mathbf{w}$ for some $a_1, \ldots, a_k \in \mathbb{R}$." It is helpful to note that, because T is onto, $\mathbf{w} = T(\mathbf{v})$ for some $\mathbf{v} \in V$. Now, what can you do with $\mathbf{v}$...

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Remember:

From now on, you can use a computer (http://www.wolframalpha.com is one option) to perform your row reductions as long as you clearly state what you have done.

§4.7 #10, 12

§4.9 #10, 34, 35, 36

§6.1 #2, 3, 4, 5, 8(c), 11(c), 12(b), 13(b), 15, 28
For #2–5, if a function is linear, you do not need to explain why. However, for each function that is not a linear transformation, you **MUST** explain why it is not.