Linear Algebra
MATH 224W – Spring 2016

Week 13: Kernel and range, matrix of a linear transformation, eigenvalues and eigenvectors

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Writing Assignment #12 **due Friday, Apr. 29**

AP #1 Let V and W be vector spaces, and assume that $L : V \to W$ is a linear transformation. Prove that

$$\dim(\operatorname{range} L) \leq \min(\dim V, \dim W).$$

(Here $\min(x, y)$ denotes the minimum of the two values x and y.) *Hint: Range-Kernel Theorem.*

AP #2 Let V and W be vector spaces with $\dim V = \dim W$, and assume that $L : V \to W$ is a linear transformation. Prove that L is one-to-one if and only if L is onto.
Hint: Range-Kernel Theorem. Don't forget that this is an if and only if statement.

AP #3 Let V be an n-dimensional vector space. Assume that $L : V \to V$ is a linear transformation such that $L(L(\mathbf{v})) = \mathbf{0}$ for every $\mathbf{v} \in V$. Prove that

$$\dim(\operatorname{range} L) \leq \frac{n}{2}.$$

Hint: Range-Kernel Theorem, but first show that $\operatorname{range} L \leq \ker L$.

Homework #12 **due Thursday, Apr. 28**

§6.2 #1, 2, 4, 6, 16, 25, 26

§6.3 #8(a)(b), 10(b), 22(a)(b)