Linear Algebra
MATH 224W – Spring 2016

Week 5: Solving linear systems and elementary matrices

Writing Assignment #4 **due ~~Monday, Feb. 15~~ Wednesday, Feb. 17**

Pg. 81 #18
Hint: In this problem you are assuming that an equation is true for *every* n-vector $\mathbf{x}$, so in particular it is true for the vectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$ where

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ldots, \mathbf{e}_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

For example, if you plug $\mathbf{e}_1$ in for $\mathbf{x}$, you get a true statement. What does it tell you?

AP #1 Rewrite the statement below as a universally quantified implication; that is, rewrite it in the form "$\forall$??? $\in$??? [(???) $\implies$ (???)]". Then **prove it using the direct method**.
"The sum of two even integers is even."

AP #2 Prove the following statement, which is an implication, by **proving the contrapositive**.
"For all $x \in \mathbb{R}$, if x is positive and irrational, then $\sqrt{x}$ is also irrational."

AP #3 Give a **proof by contradiction** of #4(d) on page 80.

AP #4 Determine if implication is associative. That is, **prove or disprove** the following: for all statements p, q, and r,

$$(p \implies q) \implies r \equiv p \implies (q \implies r).$$

Homework #4 **due Thursday, Feb. 18**

§2.1 #1(b), 4, 7, 8

§2.2 #2, 4, 8

AP #1 Let $P(x, y)$ denote the formula $x \geq y$; that is, $P(x, y)$ is interchangeable with "$x \geq y$." Also, let $\mathbb{N}$ denote the set of natural numbers (nonnegative integers), i.e. $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$. Determine whether the following are true or false; justify your answers!
(a) $\forall x, y \in \mathbb{N}[P(x, y)]$
(b) $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}[P(x, y)]$
(c) $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}[P(x, y)]$
(d) $\exists x, y \in \mathbb{N}[P(x, y)]$

AP #2 Write down **the negation of each of the following statements** in such a way that *negation symbols only appear next to the predicates* p, q, or r.
(a) $\exists x \in \mathbb{R}[(\sim p(x)) \wedge q(x)]$
(b) $\forall x \in \mathbb{R}[p(x) \implies [\exists y \in \mathbb{N}[q(x, y) \wedge r(x, y)]]]$

(c) $\exists x, y \in \mathbb{N}[P(x, y)]$