Linear Algebra
MATH 224W – Spring 2016

Week 6: Determinants

Writing Assignment #5 **due ~~Monday, Feb. 22~~ Wednesday, Feb. 24**

§2.3 #24, 25
There are many ways to approach #25, so do not feel that you must to use Theorem 2.9, as the hint suggests.

AP #1 **This is not a proof, but please still type it up (carefully).** Rewrite the following sentence in symbolic logic notation; $\mathcal{F}$ denotes the set of all functions from $\mathbb{R}$ to $\mathbb{R}$. Think carefully about the placement of quantifiers! *Hint: This statement is an implication!*

> For all $a, b \in \mathbb{R}$ with $a < b$ and any function $f \in \mathcal{F}$ that is continuous on $[a, b]$, there is some $c \in [a, b]$ such that $f(c) \leq f(x)$ for all $x \in [a, b]$.

AP #2 Let A be any $m \times n$ matrix, and assume that $\mathbf{y}$ is a solution to the linear system $A\mathbf{x} = \mathbf{b}$. If $\mathbf{z}$ is another solution to $A\mathbf{x} = \mathbf{b}$, show that there exists a $\mathbf{w} \in \mathbb{R}^n$ such that $\mathbf{z} = \mathbf{y} + \mathbf{w}$ *and* $\mathbf{w}$ is a solution to the homogeneous system $A\mathbf{x} = \mathbf{0}$. *Hint:* $\mathbf{z} = \mathbf{y} - \mathbf{y} + \mathbf{z}$.

AP #3 Prove or disprove: If A and B are invertible $n \times n$ matrices, then $(A + B)$ is invertible.

Extra Credit (*Note: I will not give any hints about extra credit problems.*) Call an $n \times n$ matrix A *nilpotent* if $A^k = 0$ for some positive integer k. Prove that if A is an $n \times n$ nilpotent matrix, then $I - A$ is invertible (where I is the $n \times n$ identity matrix).

Homework #5 **due Thursday, Feb. 25**

§2.2 #12, 14, 32
Hint: For #12, read the hint for #10. For #32, make sure to solve the system you create and find the polynomial $p(x)$.

§2.3 #2, 8, 10(a)(c), 18, 20

§3.1 #2, 4(b), 6(b), 8, 12(a)(b)