Linear Algebra
In Class—Logic

Directions: Make sure you have read the notes on mathematical logic by Professor Gibbons up through Theorem 1.16; the notes can be found in the "Documents" section of our Blackboard page. Discuss the following questions as a class and (as a class) **insert your answers below**.

1. Determine if each of the following are *statements* (as defined in the notes); make sure to justify your answers.

   (a) "Hamilton College is in Iowa."

   (b) "Determine if $2 + 3 = 4$."

   (c) "If this sentence is a statement, then it is a false statement."

2. (See Example 1.9 and Definition 1.10 in the notes.) Complete the following truth table and explain why it proves that

$$[(p \wedge q) \implies r] \equiv [(p \wedge (\sim r)) \implies (\sim q)].$$

p	q	r	$p \wedge q$	$p \wedge (\sim r)$	$(p \wedge q) \implies r$	$(p \wedge (\sim r)) \implies (\sim q)$
T	T	T	T			
T	T	F	T			
T	F	T	F			
T	F	F	F			
F	T	T	F			
F	T	F	F			
F	F	T	F			
F	F	F	F			

Explanation:

3. In the last part, you proved that $(p \wedge q) \implies r$ is logically equivalent to $(p \wedge (\sim r)) \implies (\sim q)$ using a truth table. Now prove the same thing by using the laws from the notes (De Morgan's Laws, Law of Double Negation, Law of Implication) together with the fact that disjunction and conjunction are both associative and commutative, e.g $(p \vee q) \vee r \equiv p \vee (q \vee r)$ and $p \vee q \equiv q \vee p$. (The associativity of disjunction and conjunction is covered on the next homework assignment.) I'll get you started. . .

$$(p \wedge q) \implies r \equiv (\sim (p \wedge q)) \vee r \qquad \qquad \text{Law of Implication}$$
$$\equiv ((\sim p) \vee (\sim q)) \vee r \qquad \qquad \text{De Morgan's Laws}$$

$$\equiv (p \wedge (\sim r)) \implies (\sim q)$$

4. Determine if the following implications are true or false.

   (a) "If $2 + 3 = 4$, then Josh is a Martian."

   (b) "If Josh proved a difficult theorem last Friday, then Commons served pizza for lunch last Friday."