Asymmetric Information, Dynamic Information Production and Initial Public Offerings

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Abstract: We present an information-theoretic model of IPO pricing in the presence of adverse selection and multiple trading periods. Initially investors produce information to reduce the information asymmetry and are compensated by the owner who hires a syndicate of underwriters for the IPO process. Some informed analysts evaluate the firm's prospects in the subsequent periods as new information arrives to market. By incorporating future uncertainty, subsequent information revelation and the actions of the agents, the model is able to explain not only why firms going public are underpriced but also why, on average, they underperform in the long run.

Key Words: Initial public offering, information asymmetry, Bayesian equilibrium

JEL Classification: G32, C11, C61, D81, D82

I Introduction

Initial Public Offering (IPO) is a process that makes a young firm become publicly traded corporation and helps funding available for initial investment, expansion, and other types of operating activities. It is also a process that helps original owner-manager sell her firm in a strategic way that maximizes her final wealth. However, asymmetric information about the quality of a young firm to outside investors poses an adverse selection problem that increases the idiosyncratic risk of their investment and may require higher risk premium to justify their investments (Ritter (1984), Megginson and Weiss (1991), Ljungqvist and Wilhelm (2003), and others). The presence of multiple quality firms in the financial market, in addition to the absence of full information of its own quality, poses an additional hindrance in selling a high quality firm at its fair value due to the existence of moral hazard problem among firms, where lower quality firm can mimick the higher quality firm. Existing literature show that this information asymmetry is especially problematic for a new firm (private firm) going public for raising capital or other reasons such as diversification benefit (Leland and Pyle (1977)), external monitoring (Bolton and Von Thadden (1998)), gleaning information from public (Beneviste and Spindt (1989), Dow and Gorton (1997), Habib and Ljungqvist (1998), Subrahmanyum and Titman (1999), and Maug (2001)), and avoiding high cost of going public through venture capitalists (Chemmanur and Fulghieri (1999)), among others.

Without information revelation, firms face moral hazard problem and investors encounter adverse selection problem. To resolve both problems, an obvious incentive compatible strategy for the owner-manager of a higher quality firm would be to reveal information. Since it is impossible to produce enough information to completely reveal the true value of the firm at any point in time, one effective strategy for the owner-manager would be to invite, at a cost, a certain group of investors (say, analysts and underwriters), who are endowed with technological resources and capacity to engage in information production, to process and evaluate information, and to partially reveal the quality type of the firm [it is supported by both theoretical and empirical literature, for evidence, please see Chemmanur (1993), Benveniste and Spindt (1989), Benveniste and Wilhelm (1990), Spatt and Srivastava (1991), Busaba, Benveniste and Guo (2001), Sherman and Titman (2002), and Cornelli and Goldreich (2001, 2003), among others]. In addition, since firm has a longer life, the owner-manager's strategic compensation package (such as a flat fee (cash) and a certain number of shares at discounted price) to these analysts should entice them (underwriters/analysts) to further continue engaging in information production without additional compensation (for various reasons, such as continue to invest in the firm or invest more in the firm, support the firm for future business, etc.) to enhance further sequential information revelation in subsequent period. It would also entice other outside analysts (who could not undertake the business at the IPO process but is interested in investing in such firm for own portfolio management, supporting clients' investments etc.) to engage in information production to sequentially reveal the true quality of the firm. Empirical literature is quite rich in the subsequent information production that address the likeliness of following up with recommendations after the quiet period of the IPO by the underwriting firms and others ((Bradley, Jordan, and Ritter (2006), Adam, Slovin, and Sushka (2005), Houston, James, and Karceski (2006), James and Karceski (2006), and Lin, McNichols, and O'Brien (2004)). The new information revealed to the public at the time of IPO by underwriters/analysts should help reduce the information asymmetry about the firm and increase investors' (including new investors/analysts) interest in engaging into further information production as they invest in the firm's security at the time of IPO or become interested in investing in the firm's security after IPO. This initial information production and investors' subsequent interest in further information production would help owner-manager maximize her wealth by strategically choosing a fraction of share to sell at the time of IPO and the remaining shares at its fair value in the future.

The underpricing phenomenon in IPO is first found in Mcdonald and Fisher (1972), Logue (1973) and Ibbotson (1975). Among the empricial literature, in explaining this phenomenon, Ritter (1984 and 1991) and Laughran and Ritter (1995), Ibbotson, Sindelar, and Ritter (1988) identify underpricing as a response to the asymmetric information, Rajan and Servaes (1997) justify underpricing as a means to an increased analysts following the issue after the IPO, and Demers and Lewellen (2003) explain underpricing as a substitute for conventional marketing and publicity techniques. There is also an extensive theoretical literature in explaining underpricing phenomenon. Ross (1977), Leland and Pyle (1977), Allen and Faulhaber (1989), Welch (1989) and Grinblatt and Hwang (1989) apply signaling model (underprice as a signal of good quality), Campbell and Kracaw (1980) model information production by financial intermediaries as ways of dealing with information asymmetry in the equity market, and Chemmanur (1993) models information production for underpricing as a means to generate publicity about the firm making the IPO. and Rajan and Servaes (1997) justifies underpricing as a means to an increased analysts following the issue after the IPO.

While the stock price run-up in the secondary market justifies the "underpricing" phenomenon, financial economists are puzzled by the long run "underperformance" phenomenon that exists in the empirical research. Firms that are underpriced in the IPOs, on average, underperform in the long run. Ritter (1991) and Loughran and Ritter (1995) detect this phenomenon and propose that the severe underperformance of IPOs imply that investors may systematically be too optimistic about the prospects of firms that are issuing equity for the first time. Ali (1997), Loughran and Ritter (1997), and Rajan and Servaes (1997) report that the post-issue earnings forecasts are systematically too optimistic. Healy and Palepu (1990), Brous (1992) and Jain (1992) find that analysts earning revisions forecasts are very minimal when companies announce stock issues. Loughran and Ritter (1997) observe subsequent deteriorating performance after IPO. One interpretation of their findings is that firms deliberately and successfully manage to improve earnings before going public to mislead investors. Teoh, Welch and Wong (1997) and Rangan (1998) confirm that issuers with high levels of discretionary accruals (which boost earnings relative to cash flows) have the worst subsequent stock returns. Brav and Gompers (1997) reports that the long run underperformance usually occurs in small firms (with high market-to-book stocks) that are not backed by venture capitalists. The conflicts of interest between issuers and the intermediaries are also mentioned in the literature, (see Jenkinsen and Ljungqvist (2001)). This long run under performance phenomenon strikingly implicates the importance of re-evaluation of issuing firm's future prospects (Ritter and Welch (2002). It also justifies that the secondary stock price after the IPO may incorporate the future uncertainty, and any unexpected change within the firm, industry, or in the economic cycle may have some impacts on the stock price. However, theoretical literature is absent in capturing the long run underperformance phenomenon of the IPO firms in subsequent periods. We attempt to develop a theoretical multi-period dynamic model close to the spirit of Chemmanur (1993) that would capture both underpricing and underperformance phenomena.

In an asymmetric information environment, Chemmanur (1993) proposes strategic pricing and allocation of share (as means of compensation for investors producing information that maximizes final wealth to owner-manager) among a group of investors for producing information at IPO that should serve as "publicity" to entice further information production after IPO so that the secondary market price is reflective of true value of the firm. Although the initial spirit of Chemmanur (1993) model is based on the hypothesis that "underpricing" generates publicity about the firm making the IPO and induces investors to learn more about the firm, this spirit of desire to learn more about a firm by other investors after the IPO has not been addressed completely in the model leaving the possibility of extending his model in explaing also the long-run underperformance for future research.

Most of the earlier theoretical literature, including Chemmanur (1993), model the IPO decision as a single shot which ignores the ability of the owner manager to dynamically signal its type to investors. There is an emerging literature that has begun to model the IPO decision of the firm in a dynamic framework. Benninga, Helmantel, and Sarig (2005) study the timing dimension of the decision to go public by examining the possibility of privatizing publicly traded firms. Pastor and Veronesi (2003) incorporate the aggregate productivity shocks and aggregate uncertainty about firm level productivity in a real option framework to go public. Dugger (2003) examines the strategic decision to go public as a real option exercise in a framework of asymmetric information. Idiosyncratic risk creates an incentive for private owners to sell out to investors in a public offering. However, asymmetric information about the type of the firm creates an incentive for high type firms to engage public investors in a learning process before entering the IPO market. We attempt to extend these theoretical models by introducing the necessity of further information production in the post-IPO market to explain not only the underpricing phenomenon but also the long-run underperformance phenomenon in a multi-stage setting. Further information production by analysts is very common in financial markets around the world. Empirical literature shows that underwriters who bring firm into IPO make further recommendations after the quiet period and other analysts also cover the firms after IPO (((Bradley, Jordan, and Ritter (2006), Adam, Slovin, and Sushka (2005), Houston, James, and Karceski (2006), James and Karceski (2006), and Lin, McNichols, and O'Brien (2004)).

We consider a three-period dynamic model, where insider goes public at time "0" and reveal information in both periods 0 and 1. Investors (including those who participate in initial information production) further update information at the end of date 1. By taking this dynamic structure into account, the owner-manager can decide whether to sell her residual share holdings in the secondary market in period 1 or delay. This feature of the model should capture why an undervalued firm does not issue more equity untill the full information is reflected in the stock price and hence stock price movs higher in the long run. In addition it also addresses why an overvalued firm does in the secondary market and stock price performs poorly in the long run. Another feature of this model is that all investors are dynamic and engage in further information production to reveal the quality of the firm and trades accordingly. This feature also helps explain the long run underperformance of lower quality firm and higher performance for higher quality firm.

One interesting feature of our paper is that it combines the signalling and information production models with principal-agent style model in a Bayesian setup. Unlike Chemmanur (1993), we consider 3 quality types of firms (high, medium and low) with an incomplete and imperfect prior information set such that the strategy of the owner also depends on the firm specific needs which are initially unknown by outsiders. The informed investors would only know the set of most likely strategies of a high quality firm based on their prior beliefs and experience. This allows the owner of each quality type to find the best decision in that set of (*admissible*) strategies to maximize their expected total gains from the sale of the shares by staying in the game. Following the empirical evidence on the initial optimistic beliefs of the investors, the probability of a good evaluation for each quality type would be somehow close to each other at date 0. Afterwards, it is more distinctive, depending on the new information revealed and the observations of the strategies of the owner. We characterize the equilibrium allocations and the optimal IPO price for each quality type based on the constraints (firm specific needs, the direct or indirect restrictions by the underwriters, etc.). Our numerical findings indicate that a high quality firm has a greater equilibrium IPO underpricing than the other ones, on the average, indicating a separating equilibrium. At the same time, the initial money lost during the IPO is higher when the IPO price is lower, indicating a trade-off between the initial and subsequent gains. The initial amount lost during the IPO underpricing is compensated by the future gains from the increased secondary market values as long as the owner chooses to sell the minimum amount of the shares before the terminal time. We then search for an equilibrium price that maximizes the expected total wealth of the owner. An interesting finding is that the underwriter's profit is usually maximized when the underpricing was minimal for a high quality firm and vice versa. But unfortunately, the underwriter cannot take advantage of this scenario unless he already knows the actual quality and firm-specific information with certainty. This suggests that the underpricing for a high quality firm backed by a large investment bank (with access to insider information) may not be as significant.

The strategies of the lower quality firms would be different from those of a high quality firm even with similar firm-specific prospects and constraints. But there is still a certain range of admissible strategies of high quality firms, allowing the other quality types to mimic a feasible one, supporting a partial pooling equilibrium. For example, if the initial price reduction range for a high quality firm in a specific industry is believed to be between 10% and 40%, then the owner (regardless of the quality type) should choose the IPO price in this range. For a given firm, the unconstrained underpricing level might be outside this range, for example the best choice would correspond to a 5% price reduction (e.g. for a medium quality firm). However the owner knows that she has to choose at least 10% level to please the underwriters and stay in the game. Of course, if the best price is already in the range then no adjustment is needed. The owner of a low quality firm almost always prefers the lowest amount of underpricing possible, so she would, very likely, choose a price reduction at or near 10% level in this case. The initial allocations and the IPO price chosen by the owners become a part of the subsequent information revelation, improving the quality of the analysts' evaluations at date 1.

We also specify the posterior net gains of the underwriters from the sale of all the shares of the firm for each quality type depending on the fee and contract structures. We then discuss the implications of the model for the optimization problems of both the owners and the underwriters. In particular, we discuss how the warrant rate affect the optimal choices of the agents. For the empirical results concerning warrant rates, we refer the reader to Barry, Muscarella and Vetsuypens (1991), Dunbar (1995) and Torstila (2001).

The rest of this paper is organized as follows. In section II we describe the model. The specification of the information production process and the payoffs structure is shown in section III, and we characterize the equilibrium for our model in section IV. In section V we describe the empirical implications of our model on numerical examples, relating it to the existing evidence. We conclude in section VI. A mathematical appendix and the references are given at the end.

II The Model and Problem Formulation

The problem is designed as a three-period dynamic game-theoretic model between the owner-manager (the owner, henceforth) of the firm and a syndicate of underwriters/investment banks. For the simplicity of the presentation, we assume that there is one leading underwriter who manages both the IPO process and the subsequent sales of the shares (also acting as a broker for the firm). The other underwriters and analysts are considered as informed investors at date 0. We do not explicitly model the problem of the outside (uninformed) investors. Moreover, the optimization problem of each underwriter and their agency conflicts in the syndicate are ignored. We only consider the overall gains of all the underwriters who are involved in the process of both the IPO and post-IPO sales of the firm's shares.

All the players in the game including the owner are assumed to be risk neutral[1]. The owner has perfect information about her firm's prospects, quality type and the projects while the investors have some noisy imperfect information about the firm. The leading underwriter is responsible for the information production process in a way that investors evaluate the

[1]Risk-sensitive preferences based on the utility functions can also be considered. However it makes the problem less tractable by relying too much on the (tedious) numerical computations.

firm only based on the information available to them at that particular time. When investors investigate about the firm, they receive some (noisy) additional information which is still assumed to be incomplete. The analysts re-evaluate the firm in the subsequent periods and investors incorporate that into their information set. Although the asymmetry of information between the owner and investors is reduced sequentially, information is still incomplete and it is, therefore, a model of incomplete information.

Consider that in period 0, the owner-manager has access to a project which is implemented at time 0. Assume that the project which is to be completed at date 2 offers cash flows at the end of period 2, according to the quality type of the firm. Let us denote the quality type of the firm by S that takes values in the set $\{H, M, L\}$, also indicating the quality type of the cash flow from the project which can be high (H), medium (M) or low (L). The owner knows the actual type of the project and hence the type of the firm.

Investors (outsiders) have asymmetric information about the quality type of the project and hence the type of the firm. They only know that it can be high, medium or low with certain probabilities at times $t = 0$, 1 and at the beginning of date 2. They can observe the owner's actions in each period (although not always perfectly) when the owner decides to sell some fraction of the remaining ownership. At the end of the period 2 ($t = 2+$), the quality type of the firm is known by the leading underwriter and hence by all investors. We denote the value of a firm with quality type S at time t by V_t^S, for $t = 0, 1, 2$ and $S = H, M, L$. When the superscript S is not included, it represents a random variable V_t, for $t = 0, 1, 2$. However, we sometimes drop S when the quality type is given or clear from the context, for the simplicity of the presentation. In the presence of asymmetric information, investors value the firm, by assigning prior probabilities for each type, based on the current information set Ω_0 as follows: $P(V = V^S) = p_S$, for $S = H, M, L$ such that $p_L + p_M + p_H = 1$ and $0 < V^L < V^M < V^H$. In the absence of any information production, (uninformed) investors assign the expected value of the firm as the fair market price based on the initial information set Ω_0:

$$V_0 = E[V|\Omega_0] = \sum_{S=L,M,H} p_S V^S. \tag{1}$$

Investors have the ability to produce information about the firm by investigating the firm at any period. If information is produced, the information asymmetry is reduced even further between the investors and the owner. Information production is assumed to be costly at period 0. Because of the information asymmetry, the owner has to invite some informed investors to produce information in order to complete the IPO in period 0. Investors are assumed to bid for shares if they receive good evaluation on the firm. The owner compensates only the investors who produce information and bid for shares, by offering equity at a lower price. We assume that the market for investment in securities is competitive, and for simplicity, interest rate is equal to zero. It is also assumed that if an investor receives a good evaluation, he bids for the entire block of shares offered. This assumption is made to show that popular issues are over subscribed and deliver after-market publicity among the investors in the financial market. The information production process is organized by the (leading) underwriter who also manages the distribution of the shares among the bidders. He collects the fees from the owner for his services. These fees, which are proportional to the shares sold, depend on the initial price reduction ratio V_{IPO}/V_0. For simplicity, we consider a firm commitment contract so that the unsold shares would be taken over by the syndicate of the underwriters. However, the distribution of the fees among the underwriters is beyond the scope of this work. See Torstila (2001) for an empirical study regarding the fee distribution in a syndicate.

Some degree of underpricing is encouraged by the underwriter who wants to complete the IPO process successfully and reduce the effort costs related to the distribution of the shares to the investors and the risks regarding the actual quality type (including the reputation costs for incorrect evaluations or recommendations). On the other hand, the underwriter doesn't want the price to be too low since it may conflict with the owner's (his customer's) preferences and can also reduce the amount of fees collected, which are proportional to the share prices. Let f_t denote the fee rate at date t for $t = 0, 1$ and 2. The rates which include the IPO fees, transaction costs and other brokerage fees depend on the information available at time t, for each quality type S. The initial rate f_0 is usually larger due to the IPO costs (advertising, compensation for the analysts, official procedures,...), and the extent of the initial uncertainty involved. This rate, which usually clusters around 7% for US IPOs and is a bit smaller in European IPOs (Chen and Ritter, 2000), may also depend on the initial demand for the shares of the firm, the size of the IPO and the level of the IPO underpricing (e.g. a decreasing function of the demand, the IPO underpricing and the IPO size). See also Jenkinson and Ljungqvist (2001) and Torstila (2001) for IPO spreads and management fees. The rates f_1 and f_2 would be relatively smaller depending on the evaluation results (as a proxy for the liquidity of the shares). We assume that the rate f_i is a decreasing function of the implied market price V_i at date i.

More precisely, we consider the following fee structures in the numerical examples: The initial IPO fee rate f_0 is given by

$$f_0 = \frac{V_{IPO}}{V_0}(f_{base} + f_{IPO}), \tag{2}$$

where f_b is the base fee, the ratio $\frac{V_{IPO}}{V_0}$ represents an incentive by the underwriter to motivate the owner in reducing the IPO price, and f_{IPO} accounts for the extra costs of the IPO process and/or for the extent of the initial uncertainty. The fee rate f_1 at time 1 is a fraction of f_{base}: $f_1 = \frac{V_{IPO}}{V_1} f_{base}$ which reflects the influence of the IPO underpricing on the post-IPO market. Finally, the uncertainty risk is minimal at date 2, and the rate f_2 is simply taken as a constant : $f_2 = f_{base}$, indicating fixed transaction/brokerage fees.

Some portion of the fees is paid by warrants (shares of the firm) and the remaining part by cash. Let l_0 and l_1 denote the proportion of the fees paid as warrants at times 0 and 1, respectively. The values of l_0 and l_1, which are determined by the underwriter, depend on the IPO price (a decreasing function of V_{IPO}) and on the market price V_i: $l_i = l\frac{V_i}{V_{IPO}}$, where for simplicity, we assume that there are two levels of l at each date 0 and 1: The minimum and maximum rates that are denoted by $l_{\min}$ and $l_{\max}$, respectively. The underwriter decides the (sub)optimal value of l_0 at date 0 and that of l_1 at date 1, based on the choice of $l \in \{l_{\min}, l_{\max}\}$.

The number N_0 of investors available for information production depends on the information cost for each investor and on the degree of price reduction (between the initial market price V_0 and the IPO price V_{IPO}). Moreover, $N_0 = N_0(V_{IPO})$, is assumed to be a non-increasing function of V_{IPO} on an interval $0 < V_{IPO} \leq V_{IPO}^{up} < V_0$, where V_{IPO}^{up} is an upper bound for the IPO price which is determined by the underwriters based on their belief system at date 0. When the IPO is completed after information production at $t = 0$ and the post-IPO market price of the shares are observed, the firm's projects are evaluated by some analysts whose number depend on the post IPO returns. Owner-manager decides the IPO price (V_{IPO}) and the share proportions (Δ_0, Δ_1) to sell in a dynamic setting at periods 0 and 1, respectively, by considering all the constraints and the future reaction of the underwriter and the investors. Some of these constraints are imposed by the underwriter, for example the initial share proportion Δ_0 can only be between two specified values $\epsilon_{\min}$ and $\epsilon_{\max}$. On the other hand, there are some firm specific constraints (ϵ_0 and ϵ_1) related to the proportions of total shares corresponding to the operating costs, fund-raising, project completion, etc. Such constraints may not be directly observable by outsiders including the underwriters (in practice, they may include some random noise terms and may not even be known by the owner with certainty) until date 2. However we assume that the owner observes the actions of the underwriter.

We are now ready to specify the extensive form of the game. There are two stages at each period. In period 0, at the first stage, the owner-manager is the sole owner of her firm. Based on the actual quality type, the constraints and the contract that specifies the fee and warrant rates, she decides to sell an optimal proportion Δ_0 of shares at price V_{IPO} at date 0. Then the underwriter invites all of the potential N_0 investors to produce information about the firm. Compensation for the information production is assured through discounted price sufficient to pay off the cost of production. Investors evaluate the firm and receive additional (noisy) information that reduces their informational disadvantage with respect to the owner-manager. The outcome of the evaluation is either "good" or "bad". At the second stage, investors who receive good evaluation bid for entire shares which are allocated among the bidders (e.g. proportionately). When IPO is completed in the period 0, the related information (e.g. the number of good evaluations) is made public and is reflected in the secondary market price during the first stage of period 1.

The owner decides some (or all the remaining) proportion of the shares, say Δ_1, to sell at this price V_1^S by also taking the expected value of the price at date 2 into account. Investors observe the price V_1^S and some analysts engage in a post-IPO evaluation process in the second stage of period 1. The number N_2 of the analysts depends on V_1^S, V_{IPO} and V_0. When the underpricing or the initial price reduction is significant, it can attract more analysts to review the firm. Again each analyst's evaluation is either "good" or "bad". The underwriter collects all the evaluation results, updates the probability of quality types, and hence the firm value V_2 which sets the price for the remaining shares of the firm. The game for the owner ends when she sells all the shares and pays the relevant fees to the underwriter who, then, have access to the firm-specific information and observe the actual quality type of the project and that of the firm value V. The underwriter's total wealth consist of the total fees collected plus the market value of the warrants and shares owned at date 2 minus the total costs (including the IPO expenditures, payments to the affiliated analysts, future reputation implications of handling lower quality IPO firms, etc.).

A The owner's problem

The objective of the owner is to complete the IPO in period 0, and sell the rest of the ownership in the subsequent periods in such a way that the expected total net gains from the sales is maximized. When the owner decides the IPO price V_{IPO} and the initial share allocations (Δ_0, Δ_1) to be sold at times 0 and 1, the aggregate wealth of the owner from the sale of the shares is

$$(3) \quad W = (1 - f_0(1 - l_0))V_{IPO}\Delta_0 + (1 - f_1(1 - l_1))V_1\Delta_1 + (1 - \Delta_0 - \Delta_1 - \Delta^u)(1 - f_2)V_2$$

where

- $\Delta^u = l_0 f_0 \Delta_0 + l_1 f_1 \Delta_1$ represents the proportion of the total shares to be paid as fees to the underwriter.
- $1 - \Delta_0 - \Delta_1 - \Delta^u$ represents the remaining shares (if any) that are transferred to the underwriter at date 2.

The share proportions Δ_0 and Δ_1 have the following bounds:

$$(4) \qquad 0 < \max(\epsilon_0, \epsilon_{\min}) \leq \Delta_0 \leq \epsilon_{\max} \leq \epsilon_1 \leq \Delta_0 + \Delta_1 \leq 1 - \Delta^u$$

where

- ϵ_0 : The minimum proportion of the shares the owner needs to sell during the IPO

  ϵ_1 : The minimum proportion of the shares to be sold by time $t = 1$ (especially if the completion of the projects and investments require it).

$\epsilon_{\min}, \epsilon_{\max}$: The threshold values for the IPO proportion of shares set by the underwriter.

Remark 1. *The threshold numbers $\epsilon_{\min}$ and $\epsilon_{\max}$ may represent the limits for a typical proportion of a high quality firm's shares sold at date 0. Although the firm specific value ϵ_0 may be, in practice, higher than $\epsilon_{\max}$ for a M or L value firm (especially when they need extra cash for operating expenses), which is implicit in inequality (4), the extra cash need $\epsilon_0 - \epsilon_{\max}$ can also be obtained through borrowing (perhaps with a small additional cost). So the feasible allocations can be constrained to the inequality (4) to simplify the presentation.*

- The actual firm-specific values of ϵ_0 and ϵ_1 may not be observed by the underwriter until date 2, when the project is completed and all the shares are transferred to the underwriter.
- If the fee rates are not too large and the bound ϵ_1 is not very close to 1, then the feasibility condition $\epsilon_1 \leq 1 - \Delta^u$ on the right-side of *(4)* holds. A sufficient condition for this is that $\epsilon_1 \leq \frac{1}{1+l_{\max} f_0}$ which can easily be verified using linear programming arguments.
- The aim of the owner is to maximize her expected wealth W over all feasible V_{IPO}, Δ_0 and Δ_1 values and strategies based on the constraints given above in addition to the expected future values of the share prices. For the simplicity of the presentation, we will occasionally skip the superscript S that represents the state of the firm quality.
- We denote the optimal value of W by W^*: $W^* = \max\limits_{\Delta_0, \Delta_1, V_{IPO}} W$

The owner-manager can receive only the market's perceived value given by the equation (1) without any information production. In order to reduce the information asymmetry, the owner-manager or the underwriter invites investors who conduct an evaluation of the firm at a cost. This reduces informational disadvantages of outsiders with respect to the owner-manager. The outcome of the evaluation by an investor is either "good" or "bad" and is independent of other investors. The conditional probabilities that informed investors' evaluation is good given the true types of the firm are $P(e_0 = G|S) = \alpha_S$, for $S = L, M$ and H, with $1 > \alpha_H > \alpha_M > \alpha_L > 0$.[2] The posterior probabilities of the investors with a good evaluation are derived using the Bayesian approach:

$$P(S|e_0 = G) = \frac{P(e_0 = G|S)P(S)}{P(e_0 = G)} = \frac{\alpha_S p_S}{\sum\limits_{S'=L,M,H} \alpha_{S'} p_{S'}}. \tag{5}$$

Now let X_0 denote the random variable for the number of informed investors who have a good evaluation. If it is observed to be $X_0 = r_0$, then the value V_1 of the firm from the perspective of an uninformed investor satisfies

$$V_1(r_0) = E[V|X_0 = r_0] = \sum_S \gamma_S(r_0) V^S \tag{6}$$

where

$$\gamma_S(r_0) = P(S|X_0 = r_0) \tag{7}$$

is the posterior probability weight for each quality type $S = L, M$ and H. The expression (6) dictates the secondary market value of the firm when the IPO is completed and the information is made public. Note that in period zero, the market value V_1 is random (even to the owner).

B The underwriter's problem.

The underwriter knows the actual value of the firm V only at the end of time 2. At the beginning of date 0, the initial price V_0 reflects all the initial information available about the quality type of the firm and therefore $E[V_1] = E[V_2] = E[V] = V_0$, from the perspective of the underwriter and investors. The aggregate revenue (gains) of the underwriters consists of the fees and the profit/loss from the sales of the shares:

$$\begin{aligned} Gains &= V_{IPO}\Delta_0(1 - l_0) + f_1 V_1 \Delta_1 (1 - l_1) + (1 - \Delta^u - \Delta_0 - \Delta_1) V_2 f_2 \\ &\quad + (1 - \Delta^u - \Delta_0 - \Delta_1)(V - V_2) + V\Delta^u. \end{aligned}$$

[2] Since the evaluations will depend on the content of the information provided by the owner, we will assume that α_S are not perfectly observed by the outsiders.

We are going to consider the following expression for the total costs of the underwriter:

$$
\begin{aligned}
Costs &= cost_0 V_{IPO}\Delta_0(1-l_0) + cost_1 f_1 V_1 \Delta_1 (1-l_1) \\
&\quad + cost_2(1-\Delta^u-\Delta_0-\Delta_1)V_2 f_2 + cost_{2+}\Delta^u V, \text{ with} \qquad (8)\\
cost_0 &= c_0\frac{V_{IPO}}{V_0},\ cost_1 = c_1\frac{V_{IPO}}{V_1}+c_2,\ cost_2 = c_2\frac{V_{IPO}}{V_2} \text{ and } cost_{2+} = c_2\frac{V_{IPO}}{V}
\end{aligned}
$$

where the expressions $cost_0$, $cost_1$ and $cost_2$ represent the proportions of the cash fees, which are held by the underwriter and go to the relevant costs at dates 0, 1 and 2, respectively. The terminal cost $cost_{2+}$ applies only to the warrants and is inversely proportional to the actual firm value V (accounting for the risk that they take in keeping the warrants). All of the cost terms are proportional to the IPO price to account for the advertising costs as well as costs related to the complaints/legal issues from the (institutional) customers. They are, however, inversely proportional to the market price of the security at that date to account for liquidity issues/future reputation costs. The net terminal wealth or profit of the underwriter is then given by W^u =Gains−Costs. The underwriter's aim is to select the pair (l_0, l_1) at dates 0 and 1 from a given set to maximize $E[W^u]$ sequentially.

III Information Production and the Firm's Value

Since the information production by the investors is costly, only some of the investors can afford to engage in this process for any given value of V_{IPO}. The strategy for investors is to compare the evaluated price with the offer price to determine whether to bid for the shares so that the expected payoff - net of the information production cost - is at least non-negative. Here, we are not going to formulate the optimization problem for the individual investors but rather consider the aggregate wealth of the underwriters/investment banks that manage the IPO process and the subsequent sales of the firm's shares. Depending on the quality type of the firm, the public information set and other constraints, the owner decides the optimal value of V_{IPO} and hence that of N_0, the number of investors to invite for the information production.

Before going into details of the second stage, we set the following standing assumptions for the rest of the paper:

Assumption 1. The underwriter invites all the potential investors to the information production at date 0 (for more accurate evaluation results) and all the evaluations are independent of each other. Afterwards, the posterior probabilities given in (7) serve as the prior probabilities at period 1: $P(S|\Omega_1) = \gamma_S$, which also determine the secondary market price V_1^S, for each quality type $S = L$, M and H.

Assumption 2. The firm specific proportions ϵ_0^S and ϵ_1^S are unknown by the underwriters/investors until date 2. However some certain bounds for ϵ_0^H and V_{IPO} exists: $\epsilon_{\min} \le \epsilon_0^H \le \epsilon_{\max}$ and $V_{IPO} \le V_{IPO}^{up} < V_0$, which can be interpreted as the subjective beliefs/past experience of the investors regarding a H-quality firm's IPO allocation and price sets. Then the inequality (4) reflects the constraints for each quality type based on both the firm-specific needs and the beliefs of the investors/underwriter(s).

Assumption 3. It is optimal for the owner of lower quality firms to participate in the information production rather than revealing the actual quality type (due to higher fees, liquidity costs, etc.). The probabilities β^S are valid only if the owner stays in the game.[3]

Assumption 4. The underwriters update their belief/information set by gathering and utilizing the evaluation results at each date. The underwriters' decision is sequential in nature. At date 0, the optimal warrant rate l_0^* is decided based on the initial information set Ω_0. Then the optimal rate l_1^* is determined at date 1 based on the update information set Ω_1.

Assumption 5. $E[V_1] = V_0$ from the perspective of underwriters/uninformed investors. More generally, the price process $\{V_0, V_1, V_2\}$ satisfies a *martingale* property: $E[V_i|\Omega_j] = V_j$, for $0 \leq j \leq i \leq 2$. This means that the information available at the beginning of each period sets the expected price for future periods.

Assumption 6. The initial equilibrium allocation Δ_0^* and the equilibrium price V_{IPO}^* by the owner become a part of the subsequent information revelation at date 1 and is reflected in the probabilities β_S of a good evaluation for each quality type S. The underwriters' valuation is based on the information that is made public at any particular time.

Assumption 7. The initial evaluation probabilities α_S or total proportion Δ_1 of shares sold at date 1 may not be observed by the investors/underwriters perfectly until date 2 (the owner can potentially sell the shares to various underwriters/investors at the price V_1^S until date 2).

Assumption 8. The number N_1 of analysts to evaluate the performance and the quality of the projects at date 1 depends on V_0, V_{IPO} and V_1. The evaluation by each analyst at date 1 is independent of other analysts. The outcome of the evaluation is either "good" ($e_1 = G$) or "bad" ($e_1 = B$) with conditional probabilities $\beta_S \triangleq P(e_1 = G|S)$, for $S = L, M$ and H, satisfying $1 > \beta_H > \beta_M > \beta_L > 0$. Moreover, $\beta_H > \alpha_H$ and $\beta_L < \alpha_L$, meaning that the analysts have a higher (lower, respectively) probability of assigning a good evaluation to a H (L, respectively) quality firm at time 1 than at time 0.

Assumption 9. After the number of investors who give a good evaluation in period 1 is made public, the secondary market price V_2^S is determined and shared by all investors. However the actual price V^S is only known after the underwriters/invetment banks buy the firm's remaining shares, the projects are completed and the actual quality type is made public.

Using the notation of (6) and (7), we have the following result for the secondary market price of the firm.

Lemma 1 *Assume that n investors have been invited to the information production during IPO and the number X_0 of good evaluations is not observed yet. Let V_1 be the price of the firm in the secondary market (post-IPO period), T be the true type of the firm (H, M or L) that is known by the owner, and $E^T[.]$ denote the expected value operator from the owner's perspective. Then V_1 satisfies:*

(a) $E^T[V_1] = \sum_{r=0}^{n} \sum_{S} \gamma_S V^S \binom{n}{r} \alpha_T^r (1-\alpha_T)^{n-r}$, *where* $\gamma_S = \frac{\alpha_S^r(1-\alpha_s)^{n-r} p_S}{\sum_{\acute{S}} \alpha_{\acute{S}}^r (1-\alpha_{\acute{S}})^{n-r} p_{\acute{S}}}$ *is the posterior probability weight introduced in (7)*

(b) $E[V_1|X_0 = r+1] > E[V_1|X_0 = r]$, *for each* $r = 0, ..., n-1$.

[3] Otherwise, a lower quality firm may find it less costly to go with a reservation expected wealth by taking an action that reveals the actual quality type with probability one.

(c) $E^H[V_1] \geq E^M[V_1] \geq E^L[V_1]$.

Proof. See the appendix. ■

Remark 2. *Part (b) of the Lemma indicates that the secondary market price is a strictly increasing function of the "good" evaluations for any fixed number n of investors invited. Since the owner of a higher quality firm expects more "good" evaluations than the owner of a lower quality one, we get the monotonicity result of part (c): The expected price of high quality firm would be larger than that of lower quality firms.*

Investors assign the following market value for the firm at date 1 before additional information is revealed:

(9)
$$V_1 = E[V|\Omega_1] = \sum_{S=L,M,H} \gamma_S V^S.$$

The owner then decides the Δ_1 proportion of shares to sell at this price before the evaluations by analysts are conducted at date 1. Based on this information, the posterior probabilities are updated again: Assume that $X_1 = r$ of the N_1 investors gave a good evaluation. Then the value V_2 of the firm from the perspective of uninformed investors satisfies

(10)
$$V_2(r) = E[V|X_1 = r, \Omega_1] = \sum_S \gamma'_S(r) V^S$$

where $\gamma'_S(r)$ is the posterior probability weight

(11)
$$\gamma'_S(r) = P(V = V^S|X_1 = r, \Omega_1) = \frac{P(X_1 = r|V = V^S)\gamma_s}{P(X_1 = r)}$$

Lemma 2 *Assume that n analysts evaluated the firm's projects after the IPO and* X_1 *is not observed yet. Let T be the true type of the firm (H, M or L). Then the expected price of the firm (from the owner's perspective) in the secondary market at time 2 satisfies:*

(12)
$$E^T[V_2|\Omega_1] = \sum_{r=0}^{n} \sum_S \gamma'_S V^S \binom{n}{r} \beta_T^r (1 - \beta_T)^{n-r},$$

where $\gamma'_S = \frac{\beta_S^r(1-\beta_S)^{n-r}\gamma_S}{\sum_{\acute{S}} \beta_{\acute{S}}^r(1-\beta_{\acute{S}})^{n-r}\gamma_{\acute{S}}}$.

Proof. See Appendix. ■

Note that the expected value from the owner's point of view is different than the investors' valuation since she knows the quality type (S) of the firm before the period 2 when all cash flows and the final value V_2^S are realized. The owner decides in every previous period whether or not to sell equity by considering the expected future prices.

IV Equilibrium Characterization

The equilibrium analysis, as the model involves both the owner's and the underwriter's problems, is, hence, addressed in a *principal-agent* type game theoretic setting. At each point in time, the owner makes a decision that maximizes the expected value of her total net payoffs by taking the beliefs and the current information set of the underwriters into account[4]. The equilibrium concept developed throughout the paper is perfect Bayesian equilibrium for the owner and sequential Bayesian equilibrium for the underwriters. At every stage of the game, underwriter updates belief using Bayes rule and makes a decision based on the this updated information set. We define the equilibrium as a five-tuple $(l_0^*, l_1^*; V_{IPO}^*, \Delta_0^*, \Delta_1^*)$ such that (l_0^*, l_1^*) optimize the expected profit of the underwriter given the triple $(V_{IPO}^*, \Delta_0^*, \Delta_1^*)$ which maximize the expected wealth of the owner. The details will be given below backwards in time.

A Period 2

Assume that $X_1 = r_1$ of N_1 analysts give good evaluations at time 1 and this information is known to all the parties at the first stage of period 2. Then the information asymmetry is further reduced and the resulting price $V_2(r_1)$, as in (10), is the market value of the firm. The shares of the firm can be traded at this price until the end of period 2. There is no optimization problem for the owner at this time. The owner simply sells the remaining (if any left) $1 - \Delta_0 - \Delta_1 - \Delta^u$ proportion of shares at price V_2 and hence her payoff is given by the equation (3). The fees are collected by the underwriter and the game ends for the owner.

At the second stage of date 2, the underwriter and then all the investors observe the actual value V^S (hence the quality type) of the firm and the final cash flows are realized. The realized net wealth of the underwriter is then given by

$$W^u = gains - costs$$

where gains and costs are as in subsection II.B.

B Period 1

All the agents observe the market price V_1 at time 1 following the result of the initial information production. The initial optimal IPO price V_{IPO}^* and the allocation Δ_0^* (which are assumed to satisfy the initial constraints) are already known by all the parties and the probability of a good evaluation is now given by β_s for an S quality firm. It is then time for the owner to decide which fraction of ownership to sell at the prevailing market value V_1 by considering the future uncertainty and the underwriter's warrant rate decision l_1.

The owner knows that a certain number N_1 of analysts evaluate the firm and the new price will be a direct result of the evaluations. The probability of obtaining a particular realization of $X_1 = r$ for a given N_1, conditional on the firm types are computed using the binomial distribution:$P(X_1 = r|S) = \binom{N_1}{r}\beta_S^r(1-\beta_s)^{N_1-r}$ for $S = L, M$ and H. The owner can obtain the posterior probabilities γ_S' of the quality types for each $r = 0, 1, ..., N_1$ and hence the expected value of the firm as in Lemma 2. As the quality of the firm improves, the expected number of the good evaluations increases. So it is to the advantage of the owner of a high quality firm to sell a small proportion of shares at this time and wait for the terminal time 2

[4]This setup can also be considered a Stackelberg game in which the owner is the leader of the game while the underwriter is the follower.

to distribute the remaining shares. The realized value of X_1 will be known at the end of the period. We have the following results for the problems of the owner and the underwriter:

Lemma 3 *Assume that the number X_1 of good evaluations at date 1 is made public. Then,*
(a) ***The underwriter's problem at date 1****. The optimal choice of l_1 is given by*

$$l_1^* = \begin{cases} \frac{V_1}{V_{IPO}} l_{\max}, & \text{if } (c_2 - f_b)V_1 - (c_2(-f_b) - c_1)V_{IPO} > 0 \\ \frac{V_1}{V_{IPO}} l_{\min}, & \text{if } (c_2 - f_b)V_1 - (c_2(-f_b) - c_1)V_{IPO} < 0 \end{cases}$$

where c_1 and c_2 are as in the cost functions $cost_1$ and $cost_2$ of subsection II B.
(b) ***The owner's problem at date 1****. Let the pair $(l_0, l_1) = (l_0^*, l_1^*)$ be the optimal selection for the underwriter's problem, which is observed by the owner. Then the optimal proportion Δ_1^* of the owner is determined by the 3 cases below:*

- *Case 1: If $V_1 < \frac{1+l_1 f_1}{1-f_1(1-l_1)}(1-f_b)E^T[V_2|\Omega_1]$, then $\Delta_1^* = \epsilon_1 - \Delta_0^*$.*
- *Case 2: If $V_1 > \frac{1+l_1 f_1}{1-f_1(1-l_1)}(1-f_b)E^T[V_2|\Omega_1]$, then $\Delta_1^* = \frac{1-(1+l_0 f_0)\Delta_0^*}{1+l_1 f_1}$.*
- *Case 3: If $V_1 = \frac{1+l_1 f_1}{1-f_1(1-l_1)}(1-f_b)E^T[V_2|\Omega_1]$, then Δ_1^* can be any value between $\epsilon_1 - \Delta_0^*$ and $\frac{1-(1+l_0 f_0)\Delta_0^*}{1+l_1 f_1}$.*

C Period 0

Following the time line through a backward procedure, the owner's strategic decision to sell some proportion of the firm is at hand in period zero. At this time, the owner has to decide what fraction Δ_0 of equity sales, at an equilibrium price V_{IPO}, would maximize the combined proceeds of all sales. She knows that after IPO is complete, the secondary market price V_1 is going to be influenced by the evaluation results. The owner has to decide a level of Δ_0 and an initial price level V_{IPO} so that she can maximize her combined expected payoffs from the total sales of the firm.

In period 0, when the owner (or the underwriter on her behalf) approaches N_0 investors to evaluate her firm, the number X_0 of investors who assign good evaluations is a random variable to her. The secondary market price of the equity is going to be guided by the investors' evaluated price. The probability of obtaining a particular realization r_0 of X_0 for a given N_0, conditional on the firm type S is given by $P(X_0 = \delta_0|S) = \binom{N_0}{r_0}\alpha_S^{r_0}(1-\alpha_s)^{N_0-r_0}$ using (conditional) Binomial distribution. The actual post-IPO market price V_1 is random before the IPO is completed and can be higher or lower than the owner's expectation. The owner can exercise her option to sell some or all of her remaining shares at this market price to maximize her total proceeds. So the optimization problem to be solved at time 0 involves finding the optimal values $\Delta_0^*, \Delta_1^{*,0}$ and V_{IPO}^* (and hence the amount of information cost, in other words, how many investors N_0 to invite) that maximizes the expected terminal wealth from the sale of all shares of the firm.

The initial optimal allocations $(\Delta_0^*, \Delta_1^{*,0})$ and the optimal IPO price V_{IPO}^* are obtained simultaneously by considering the actual quality type, all the constraints, the reactions of the underwriter and the investors, and future random variables regarding the values of the firm and the number of good evaluations both at times 0 and 1. Namely,

$$(13) \qquad \begin{aligned} W^* &= \max_{\Delta_0,\Delta_1,V_{IPO}} \{V_{IPO}\Delta_0(1-f_0(1-l_0)) + E^T[V_1]\Delta_1(1-f_1(1-l_1)) \\ &\quad +(1-\Delta_0-\Delta_1-\Delta^u)E^T[(1-f_2)V_2]\} \end{aligned}$$

where $V_{IPO} \leq V_0$, $\Delta^u = l_0 f_0 \Delta_0 + l_1 f_1 \Delta_1)$ represents the proportion of shares to be paid as fees to the underwriter, the superscript T indicates the true quality type and $(\Delta_0, \Delta_1, \Delta^u)$ satisfy the inequality (4). The optimization problem is linear in (Δ_0, Δ_1) but highly nonlinear and implicit in V_{IPO} since the future expected prices and the optimal values of the pair (l_0, l_1) which are determined by the underwriter also depend on V_{IPO}. In practice, the problem can only be solved numerically by considering discrete values of V_{IPO} over an interval below V_0. Then $E^T[V_1]$ and $E^T[V_2]$ from the perspective of the owner are computed for each fixed V_{IPO}. The equilibrium values are obtained sequentially: At time 0, we can only obtain the optimal values Δ_0^* and V_{IPO}^* (numerically). The actual optimal value Δ_1^* is obtained at time 1. We denote the estimate of Δ_1^* that is obtained at time 0 by $\Delta_1^{*,0}$ to distinguish between these two proportions (although they are very likely to be the same).

For each fixed value of $V_{IPO} = v$, the expectations $E^T[V_1]$ and $E^T[V_2]$ depend on v implicitly. Moreover, the underwriter's decision (l_0, l_1) and the fee rates also depend on v. Hence, given the best response $(l_0, l_1) = (l_0^*, l_1^*)$ of the underwriter, the expected value W can be written as a function of (v, Δ_0, Δ_1) as follows:

$$(14) \qquad \begin{aligned} W &= E^T[(1-f_2)V_2] + \Delta_0\{v(1-f_0(1-l_0)) - E^T[V_2(1-f_2)](1+l_0 f_0)\} \\ &\quad +\Delta_1\{E^T[V_1](1-f_1(1-l_1)) - E^T[V_2(1-f_2)](1+l_1 f_1)\} \end{aligned}$$

where the dependence of each term on v is implicit in the equation.

Notation: For each fixed v, define

$$\hat{W}(v) \triangleq \max_{(\Delta_0,\Delta_1)} f(v, \Delta_0, \Delta_1) = f(v, \Delta_0^*(v), \Delta_1^{*,0}(v)).$$

Then the optimal initial allocations $\Delta_0^*(v)$ and $\Delta_1^*(v)$ can be obtained by a simple linear programming approach for each v. Finally, the optimal values of v and W are obtained numerically by solving $W^* = \max_{v \leq V_0} \hat{W}(v)$ over a set of discretized values of v. In general, the equilibrium results depend on the constraints and the assumptions on the parameters. The following Lemma summarizes all the possible cases with strict inequalities that result in unique solutions:

Lemma 4 *Let $V_{IPO} = v$ be a feasible IPO price.*

(a) ***The underwriter's problem at date 0****. The optimal choice of l_0 is given by*

$$l_0^* = \begin{cases} \frac{V_0}{v} l_{\max}, & \text{if } V_0(1-f_b) - v(1 - c_0 \frac{v}{V_0} + c_2(1-f_b)) > 0 \\ \frac{V_0}{v} l_{\min}, & \text{if } V_0(1-f_b) - v(1 - c_0 \frac{v}{V_0} + c_2(1-f_b)) < 0 \end{cases}$$

(b) ***The owner's problem at date 0****. Given the optimal strategy $(l_0, l_1) = (l_0^*, l_1^*)$ of the underwriter, the initial optimal proportions $\Delta_0^*(v)$ and $\Delta_1^{*,0}(v)$ satisfy the followings:*

(i) If $\max(\frac{1-f_0(1-l_0)}{1+l_0 f_0}v, \frac{1-f_1(1-l_1)}{1+l_1 f_1}E^T[V_1(v)]) < (1-f_b)E^T[V_2]$, *then* $\Delta_0^*(v) = \epsilon_0$ *and* $\Delta_1^{*,0}(v) = \epsilon_1 - \epsilon_0$.

(ii) If $\frac{1-f_1(1-l_1)}{1+l_1 f_1}E^T[V_1(v)] < (1-f_b)E^T[V_2] < \frac{1-f_0(1-l_0)}{1+l_0 f_0}v$, *then* $\Delta_0^*(v) = \epsilon_{\max}$ *and* $\Delta_1^{*,0}(v) = \epsilon_1 - \epsilon_{\max}$.

(iii) If $\frac{1-f_0(1-l_0)}{1+l_0 f_0}v < (1-f_b)E^T[V_2] < \frac{1-f_1(1-l_1)}{1+l_1 f_1}E^T[V_1(v)]$, *then* $\Delta_0^*(v) = \epsilon_0$ *and* $\Delta_1^{*,0}(v) = \frac{1-(1+f_0 l_0)\epsilon_0}{1+l_1 f_1}$.

(iv) If $\min(\frac{1-f_0(1-l_0)}{1+l_0 f_0}v, \frac{1-f_1(1-l_1)}{1+l_1 f_1}E^T[V_1(v)]) > (1-f_b)E^T[V_2]$, *then* $\Delta_0^*(v) = \epsilon_{\max}$ *and* $\Delta_1^{*,0}(v) = \frac{1-(1+f_0 l_0)\epsilon_{\max}}{1+l_1 f_1}$.

Remark 3. *When the inequalities are not strict, there may be multiple solutions for the optimal allocations.*

Proposition 1. *An equilibrium* $(l_0^*, l_1^*; V_{IPO}^*, \Delta_0^*, \Delta_1^*)$ *always exists. The owner's wealth* W^* *corresponding to the optimal choices* $(V_{IPO}^*, \Delta_0^*, \Delta_1^*)$ *always optimizes her realized total wealth from the sales of the shares. However the underwriter's realized profit may not be always optimal.*[5]

V Empirical Implications

The empirical implications of the model depends heavily on the firm-specific values and on the initial beliefs of the underwriter(s). When Δ_0 is not announced publicly before IPO, the optimal initial allocation Δ_0^* depends on the actual quality type of the firm so that we actually have a *separating* equilibrium. However, the investors cannot perfectly identify the quality type from these allocations. It only helps them to improve their ability to evaluate the firm more accurately and update their beliefs (probabilities for each type)/information set. On the other hand, the owner would take advantage of the investors' inferior information before announcing the IPO price and the proportion of shares to be sold in IPO. Therefore a partial imitation of the strategy of a high quality company by the owners of low or medium quality firms at the same time hiding the quality type in the beginning is one of the main contributions of our paper.

A Numerical Examples

We now provide some numerical examples to explain the empirical implications of the model. The agents' problems are solved at time 0 as follows: For each IPO price V_{IPO}, the underwriter's best response l_0^* and the estimated value of l_1^* are computed based on the model parameters. Then the owner's optimal allocations $(\Delta_0^*, \Delta_1^{*,0})$ are obtained from the owner's perspective. Finally, the expected wealth of the owner is computed on a grid of feasible IPO prices to determine the best security price V_{IPO}^* numerically.

Example 1. *Consider the IPO of a firm in a certain industry with the actual market prices for a H, M and L value firms given by* $V^H = 25$, $V^M = 20$, *and* $V^L = 16$ *(in million dollars*

[5] *If we consider a model that allows the underwriter to know the insider information except the quality type, then it results in a pooling equilibria for all quality types.*

for a total of one million shares, say), respectively.[6] *The informed investors only know the initial probabilities about the quality of the firm, which are given to be* $p_H = 0.35, p_M = 0.40$ *and* $p_L = 0.25$*, in this case. The initial market price is then* $V_0 = 20.75$ *(a value between* V_M *and* V_H*). The firm hires an investment bank (the lead underwriter) to form a syndicate of underwriters in managing the IPO process and the subsequent sales of the shares. The threshold share proportions for IPO sales are considered to be* $\epsilon_{\min} = 0.18$ *and* $\epsilon_{\max} = 0.30$ *which also set the boundaries for the IPO allocation* Δ_0 *of a typical H-value firm (from the point of view of the underwriters/market). The underwriter also restricts the IPO price to an upper bound* $V_{IPO}^{up} = 20.5$ *based on his initial belief/information set. The conditional probabilities that informed investors' evaluation at date 0 is "good" given the true types of the firm are estimated to be* $\alpha_H = 0.80, \alpha_M = 0.70$ *and* $\alpha_L = 0.40$ *by the owner. Similarly, the conditional probabilities that informed investors' evaluation at date 1 is "good" given the true types of the firm are* $\beta_H = 0.85, \beta_M = 0.55$ *and* $\beta_L = 0.3$ *(more accurate than* α*).*

The number N_0 *of investors available at each IPO price* v *is given by a piecewise linear function as follows: At the initial share price of* $V_0 = \$20.75$, $N_0(20.75) = 20$*; then it increases by 3 investors for each price reduction of 5 cents per share until 19.30 dollars. Below that price, the number increases by 2 investors for each reduction of 5 cents until 18.55 dollars, and so on. Moreover, the number of analysts at date 1 is a non-decreasing function of the expressions* $|V_1 - V_{IPO}|$ *and* $V_1 - V_0 : N_1 = \max(N_1^{\min}, round(4\,|V_1 - V_{IPO}| + 6\,|2(V_1 - V_0) + 0.1| + 2\sqrt{1 + 2dist^2})$ where $dist = max(0, V_1 - V_{IPO})$ and $N_1^{\min} = 16$.

The fee schedule is given as follows: $f_0 = (0.05 + 0.025)\frac{V_{IPO}}{V_0}$ *so that the maximum rate becomes* 7.41% *when there is minimal reduction for the IPO (at* $V_{IPO} = V_{IPO}^{up} = 20.5$*). Moreover, the rates at dates 1 and 2 are* $f_1 = (0.05)\frac{V_0}{V_1}$ *and* $f_2 = 0.05$*. The maximum and minimum warrant rates are* $l_{\max} = 0.25$ and $l_{\min} = 0.10$.

The actual cash constraints ϵ_0 *and* ϵ_1*, which depend on the needs and the quality type of the firm, satisfy* $0.18 \leq \epsilon_0 \leq 0.30 < \epsilon_1$ *but are unknown by the underwriter at date 0. The underwriter's warrant rate selection at date 0 is based on the Lemma 4: The threshold value* $V_0(1 - f_b) - V_{IPO}(1 - cost_0 + c_2(1 - f_b))$ *is positive when* $V_{IPO} \leq 20.35$ *in the price interval. Hence* $l_0^* = \frac{V_0}{V_{IPO}} l_{\min}$ *when the IPO price (selected by the owner) is above 20.35, and it is* $\frac{V_0}{V_{IPO}} l_{\min}$ *otherwise. The total profit of the underwriters depend on the fees collected, the terminal value of the warrants and the total costs that all depend on the IPO price. We now study the optimization problem in detail for each quality type separately.*

Example 2, H-value firm: *The optimal expected gains* $E^H(W^*)$ *of the owner of a high quality firm depends on the proportions* ϵ_0 *and* ϵ_1 *as well as on the warrant allocations of the underwriter. Figure 1 shows that it tends to decrease with both parameters* ϵ_0 *and* ϵ_1*.*

We now consider some fixed firm specific proportions $\epsilon_0 = 0.20$ and $\epsilon_1 = 0.5$ *for a H-quality firm to decribe the impact of the individiual parameters. The expected secondary market prices* $E^H[V_1]$ *and* $E^H[V_2]$ *together with the expected wealth* W *as a function of the IPO price are plotted in Figure 2. It is clear that the secondary market prices tend to increase as the IPO price decreases. The optimal value (with increments of 0.05) of IPO price is* 19.35 *corresponding to the optimal expected wealth* $E(W^H) = 22.3186$ *and* $N_0 = 104$*. The owner sells* $\Delta_0^* = \epsilon_0 = 0.20$ *proportion of the shares at the price* $V_{IPO} = 19.35$*. Then after the result of the evaluations (*r*) is made public, the new market price* V_1 *is determined. The truncated*

[6] *So the share price and the firm value will be represented by the same quantity, by an abuse of notation.*

plot of the secondary market price $V_1(r)$ *and the conditional expectation* $E^H[V_2|r]$ *in Figure 3 shows how the market value gets closer to the true value of the firm based on the observed values of* r*. For a H-value firm, it is very likely that most of the evaluations would be good, e.g. above 80, and therefore the market value* $V_1 = V_1(r)$ *at date 1 would be close to* $V^H = 25$*. However if, for any reason, that is not the case so that the market value is much lower than* V^H*, the evaluations at date 1 would help bring the value closer to* V^H *again. It is clear from the figure that* $E^H[V_2|r] > V_1(r)$*, for every* r*. The reason that the graph of* $E^H[V_2|r]$ *is not monotone is the specific nature of the number* N_1 *of the analysts at date 1. Since* N_1 *is larger at the extreme values of* r *and the expected price is an increasing function of* N_1*, the expected value* $E^H[V_2|r]$ *is relatively smaller for the middle values of r. The optimal allocation* Δ_1 *of date 1 is guessed to be* $\epsilon_1 - \epsilon_0 = 0.3$ *at date 0. Consider the following scenario at date 1: Assume that* $r = 78$ *of the* 104 *invited investors assigned a "good" evaluation for the firm. Then the corresponding market price at date 1 becomes* 22.1766 *and the owner's expected market price* $E^H[V_2|r = 78]$ *at the beginning of date 2 is* 24.8378*. This implies that the owner actually goes with the allocation* $\epsilon_1 - \epsilon_0 = 0.3$ *at date 1 and sell the remaining shares at date 2.*

The underwriters/investment banks then buy the remaining shares from the owner at the observed price V_2 *at date 2 and sell them at the true value* $V^H = 25$ *after date 2.*

Analysis from the underwriter's point of view:

The underwriter has initial beliefs and noisy information about the firm's quality and relies on the evaluations by the investors. The IPO firm may be a low or medium quality firm with a good image due to the intensive advertising or aggressive performance before the IPO, or it may be a high value firm with insufficient advertising to get attention to its good projects prior to the IPO process.

The costs of the underwriter depend on the IPO price, the actual quality type, the levels of initial and subsequent underpricing (or overpricing) and the allocation amounts of the owner. The specific cost function using the notation of (8) for this example has the parameters $c_0 = 0.25$, $c_1 = 0.15$ *and* $c_2 = 0.22$*. The underwriter decides the warrant rate* l_0 *at date 0 based on* V_0 and V_{IPO} *but cannot accurately guess the rate* l_1 *of date 1 yet.*

The low IPO price $V_{IPO} = 19.35$ *and the low allocation proportion* $\Delta_0 = 0.20$ *are good signs for the firm's quality type but it is still not clear if it is due to the incentives by the underwriter or the partial imitation strategy of lower quality ones. Such a low IPO price results in a higher warrant rate and may allow the underwriter to distribute the initial shares to its favorite customers (who participate in the information production) at a lower price, decreasing the overall costs and future risks of the underwriter.*

Again, assume $r = 78$ *of the* 104 *invited investors assigned a "good" evaluation for the firm and the updated price is* $V_1 = 22.1766$ *at beginning of date 1. The updated probabilities for the quality types are roughly* $\gamma_H = 0.4353, \gamma_M = 0.5647$ *and* $\gamma_L = 0$*, (rounded to four decimal places) indicating that it is almost unlikely to be a low quality firm and is more likely to be a medium quality one (thanks to the optimism of the investors). So the more accurate evaluations by analysts at date 1 are crucial to distinguish the quality types (between H and M value firms, in this case).*

The underwriter cannot solve his maximization problem optimally due to the information asymmetry. However, his expected profit can be computed from the owner's perspective who *has access to all the information in this game. We treat this computation as the expected realized profit of the underwriter in our setup. Figure 4 indicates that the optimal IPO price for this example would be around the upper bound 20.5. Our numerical computations based on the potential range of the pairs* (ϵ_0, ϵ_1) *show that the best IPO price for the underwriter is not usually smaller than the cut-off point of the warrant rate shift, 20.35 in this case. There is always a local maximum at this point which sometimes gives the absolute maximum.*

The comparative statics of the equilibrium IPO price, owner's expected wealth and the underwriters' expected total profit versus each of the parameters ϵ_0 *and* β_H *(as a proxy of the evaluation certainty) are given in tables I and II. The results in Table I show that the agents' optimization problems have conflicting equilibrium values with respect to the initial allocation constraint* ϵ_0*. As* ϵ_0 *increases, the owner selects a higher IPO price (hence a lower level of underpricing) and predicts a smaller expected wealth. On the other hand, the underwriters' total expected profit slightly increases with* ϵ_0*. It means that the the gains from a larger allocation of IPO shares to their favorite customers together with higher fees collected from the owner dominate the costs of a higher IPO price, on the average. Furthermore, the Table II indicates that the underpricing is positively correlated with the evaluation uncertainty for a high quality firm, supporting the empirical findings regarding the relationship between (ex-ante or ex-post) uncertainty and the underpricing.*

Example 3, L-value firm: *For a low quality firm, it is reasonable to consider the case* $\epsilon_0 = \epsilon_{\max} = 0.30$ *(the optimal initial allocation is almost always* $\epsilon_{\max}$ *even though* ϵ_0 *may be smaller). The owner allocates* $\Delta_0 = \epsilon_{\max} = 0.30$ *proportion of shares at date 0 with the price* $V_{IPO} = 20.50$ *(the upper bound IPO price). The allocation at date 1 depends on the evaluation results. It is either* $\Delta_1 = 1 - \epsilon_{\max} - \Delta^u$ *which would occur only when* $V_{IPO} > 20.50$ *in this case (outside the range of the admissible IPO prices). For smaller values of* V_{IPO}*, it is* $\Delta_1 = \epsilon_1 - \epsilon_{\max}$*, which occurs especially when the fees at date 1 are sufficiently large. This indicates that the underwriter's incentives to encourage the owners in keeping some shares until the date 2 may work even for a low quality firm. The kink at* $V_{IPO} = 20.35$ *in Figure 5 is due to the transition from* $l_{\max}$ *(when* $V_{IPO} \leq 20.35$*) to* $l_{\min}$ *(when* $V_{IPO} > 20.35$*). The owner's expected final wealth* $E^L[W]$ *is increasing with both* V_{IPO} *and* l *but decreasing with* ϵ_1*. On the other hand, the underwriter's expected realized wealth* $E[W^u]$ *is not monotonic with* V_{IPO} *but increasing with* ϵ_1*. The absolute maximum of* $E^L[W^u]$ *usually occurs after the cut-off point 20.35 (at 20.40 in this case) where the warrant rate is minimum. The other local maximum point, which is attained at smaller* V_{IPO} *values, shifts to the left and occasionally is the absolute maximum for smaller values of* ϵ_1 *(Figure 6). Since the underwriters and investors are not aware of these facts, they can rarely obtain their absolute optimal equilibrium results. On the other hand, the owner can take full advantage of her informational superiority and over-optimism of the market's view of the firm at date 0. However, she looses that advantage as the information asymmetry decreases by time. Therefore, the final expected wealth* $E[W^L]$ *would be much larger than the secondary market prices at dates 1 and 2 for reasonably large IPO price levels (Figure 7). We always have* $V^L = V_{2+} \leq E^L[V_2] \leq E^L[V_1] \leq V_0$*. So the information production and the subsequent evaluations insure that the market price gets corrected gradually. Moreover,* $E^L[W]$ *could get even smaller than* $V^L = 16$ *when* V_{IPO} *is relatively small (due to the losses from the higher warrant rate). However, such small values would be optimal for neither owner nor the underwriters.*

Example 4, M-value firm. *The relevant Lemmas in the previous section covers all the* *possibilities for the allocation decision of a M-value firm. The optimal initial allocation Δ_0^* is equal to ϵ_0 for all the admissible IPO prices (it is $\epsilon_{\max}$ only at 20.60 or larger values). Moreover, the (initial) optimal allocation estimate $\Delta_1^{*,0}$ is $1 - \Delta_0^* - \Delta^u$ which depends on the warrant rate l and ϵ_0 (it is larger when $l = l_{\min}$ and ϵ_0 is smaller) and is independent of ϵ_1 in the range of the parameters considered. The expected wealth $E^M[W]$* of the *owner decreases with ϵ_0 (Figure 8(a)) while the underwriter's wealth increases (Figure 8(b)). The extent of the optimal underpricing depends on the over-optimism and the evaluation quality of the investors (captured by the probabilities α* and *β) and is usually very small. In the range of IPO prices considered, the expected secondary market price $E^M[V_1]$* is always above the actual value V^M *and mostly above V_{IPO}. However $E^M[V_2]$ gets much closer to V^M (Figure 9). We believe that such an observation would explain the long-run underperformance observations following the IPO. Although there is a run-up in the secondary market, it is mostly due to the information asymmetry and the over-optimism of the market. The optimal allocation at date 1 again depends on the incentives, fees and the observed value of the number of the good evaluations by the investors at date 0. When $V_{IPO}^* = 20.5$,* *both the owner's and the underwriter's wealth would be optimized however the underwriter wouldn't know it until date 2.*

B Model Implications

For most of the reasonable values, the following results hold:

High (H) value firm. The owner takes the full advantage of information production to reduce the information asymmetry. The secondary market price V_1^H is close to the true value V^H. Moreover, for almost all of the parameters, the part (a) of the Lemma 4 applies: $\Delta_0^* = \epsilon_0^H$ and $\Delta_0^* + \Delta_1^{*,0} = \epsilon_1^H$. The owner sells the minimum possible shares at time 0 and predicts to do the same at time 1 based on the initial information set. The actual value Δ_1^* is determined after the evaluation results at date 0 are made public. The level of the underpricing is positively correlated with the uncertainty in the evaluations at date 1. The underwriter's expected net gains are usually increasing with the IPO price. Therefore, an underwriter wouldn't be so aggressive in providing incentives for the IPO of a high quality firm to reduce IPO price significantly (when the quality is known to the underwriter). This suggests that the IPOs of high quality firms backed by large investment banks would offer a relatively lower underpricing compared to the other high quality ones. The positions of the owner and the underwriter contradict regarding the warrant rate and the initial allocation ϵ_0. The underwriter benefits from a higher warrant rate and ϵ_0 while the owner prefers a minimal warrant rate and ϵ_0.

Low (L) value firm. The owner prefers to avoid information production to preserve the information asymmetry (thanks to the current level of optimism in the market). Depending on the parameter values, the extra fees that are paid to the underwriter may force the owner to follow an imitation strategy of a H-value one in order to disguise the true type as much as possible. The owner is conservative about reducing the IPO price unless it helps to increase the warrant rate since the warrants are more valuable (as proportions of the higher share prices) initially. The secondary market price V_1 is a bit close to the true value V^L and the price gets much closer to V^L at date 2, on the average. Moreover, for almost all the parameters, the initial allocation is $\Delta_0^* = \epsilon_{\max}$ while Δ_1 depends on whether the net (of the fees) payoff at time 1 is smaller than the expected payoff at date 2, resulting in either $\Delta_1 = \epsilon_1 - \epsilon_{\max}$ or

$1 - \Delta_0^* - \Delta^u$ (the part (b) or (d) of the Lemma 4 applies).

Medium (M) value firm. The owner has a richer set of strategies, in general. The initial allocation Δ_0^* can range from ϵ_0^M to $\epsilon_{\max}$. Likewise, Δ_1 ranges between $\epsilon_1^M - \epsilon_{\max}$ and $1 - \epsilon_0^M - \Delta^u$. When the over-optimism in the current market is very significant, then a very likely price run-up in the secondary market can still indicate a false signal about the quality type. The (especially uninformed) investors would expect a continuation of the price increase in the long run. However, when the projects are getting to the end and the analysts at date have more accurate evaluations, the market value at date 2 is very close to the actual value V^M. A likely scenario is that $V_1^M > V_2^M \geq V^M$. If a majority of the firms going IPO are of M-value type, then this can be a satisfactory explanation of the long-run underperformance observations (which may not be an actual underperformance) of the empirical data.

Underwriter. The optimal value of IPO price for the underwriter may not be solved from the underwriter's perspective under this model unless the underwriter would know the probability distribution of firm specific information for each quality type or the information for just a typical firm in each category of quality types. Then the underwriter or underwriters would decide some of the parameters to force the owners in choosing the allocations/IPO price that are closest to underwriter's optimal values. One way to do this is to adjust the fee rates to cover the possible costs of the non-optimal IPO price; the other would be adjusting the warrant rates (as a control variable) for the fees accordingly. We include both of these concepts in the model and show that they provide only sub-optimal choices for the underwriter who takes the IPO price and the initial market price as a proxy for the initial warrant and fee rates. In general, the cut-off point for a higher warrant rate occurs at a lower equilibrium IPO price and the underwriter benefits from that if the firm is of a high-quality one. Since it is less likely for a low quality firm to reduce the price to that level (except that the owner may reduce it further to increase the warrant rate), the underwriter only uses the minimum warrant rate in that case. Therefore, our model captures the empirical observations that the warrants would increase the underwriters' total compensation [Torstila (2001), Barry, Muscarella and Vetsuypens (1991)] or decrease the IPO costs (Dunbar (1995)), on the average.

VI Conclusion

We model the IPO and the subsequent prices of a firm depending on a game between the owner and the informed investors in a multi-period information theoretic setting. We characterize the equilibrium allocations (Δ_0^*, Δ_1^*) and optimal IPO price V_{IPO} of the owner, and the (sub)-optimal warrant rates (l_0^*, l_1^*) of the underwriters for each quality type. We find that the owner of low or medium quality firm can take advantage of her superior information given the constraints, and she can partially imitate a potential strategy of a H-quality firm without revealing the actual quality type. The firm-specific constraints/needs are as important as the actual quality type and can help the owner in such an imitation strategy. However, the owner of a high quality firm can usually afford and benefit from a more significant underpricing/initial price reduction than the other firms. The level of the underpricing increases with the uncertainy in the analysts' evaluation accuracy. Our model results in a separating equilibria for different quality types which are not observed perfectly until the terminal time.

The underwriter's profit is not always maximized at the optimal choice of the owner. It is usually larger for the high quality firms. The optimal profit depends on some trade-off among the fees collected, the costs involved (both are proportional to IPO price) and the actual value of the warrants.

The model predicts that the market price of a firm would get closer to the actual value after evaluation by the investors/analysts and the "underperformance phenomenon" reported in empirical studies may actually be some "initial overperformance" arising from a combination of the over-optimism of the market, information asymmetry, and the conflicts between the owner and the underwriter.

Some Potential Extensions.

- An extension of this work is to allow time dependent or random quality types of the projects at each time. For example, the owner may decide (or believe) to start with a H-quality project but the result of the evaluations at date 1 may force her to switch to a M-quality project at date 1 for an optimal expected terminal wealth.

- Similarly, we can allow the accuracy of the evaluations (measured by the probability vectors α and β) to depend on the IPO price or other parameters of choice rather than taking it just as a constant.

APPENDIX. The proofs of the lemmas in the text.

Proof of Lemma 1.

(a) From the owner's point of view, the expected price will be

$$E^T[V_1] = \sum_{r=0}^{n} E^T[V_1|X=r]P^T(X=r),$$

where $P^T(.)$ is the probability measure from her perspective (given the actual quality type). Hence $P^T(X=r) = \binom{n}{r}\alpha_T^r(1-\alpha_T)^{n-r}$, and $E^T[V_1|X=r] = E[V_1|X=r] = E[V|X=r]$ which is independent of the owner's firm specific information and is given by (6). Moreover, by (7) and Bayes rule, we obtain

$$\begin{aligned}\gamma_S(r) &= P(S|X_0=r)\\ &= \frac{P(X_0=r|S)p_s}{P(X_0=r)}\\ &= \frac{\binom{n}{r}\alpha_S^r(1-\alpha_s)^{n-r}p_S}{\sum_{\acute{S}}\binom{n}{r}\alpha_{\acute{S}}^r(1-\alpha_{\acute{S}})^{n-r}p_{\acute{S}}}\\ &= \frac{\alpha_S^r(1-\alpha_s)^{n-r}p_S}{\sum_{\acute{S}}\alpha_{\acute{S}}^r(1-\alpha_{\acute{S}})^{n-r}p_{\acute{S}}}.\end{aligned}$$

(b) The identity $E[V_1|X=r+1] > E[V_1|X=r]$ holds if and only if

$$\frac{\sum_S \alpha_S^{r+1}(1-\alpha_s)^{n-(r+1)}p_S V^S}{\sum_S \alpha_S^{r+1}(1-\alpha_S)^{n-(r+1)}p_S} > \frac{\sum_S \alpha_S^r(1-\alpha_s)^{n-r}p_S V^S}{\sum_S \alpha_S^r(1-\alpha_S)^{n-r}p_S}$$

which is equivalent to expression

$$(15)\qquad (\sum_S \frac{\alpha_s}{1-\alpha_s} u_S V^S)(\sum_S u_S) > (\sum_S \frac{\alpha_s}{1-\alpha_s} u_S)(\sum_S u_S V^S)$$

where $u_S = \alpha_S^r (1-\alpha_S)^{n-r} p_S$. After writing the expressions in both sides of (15) explicitly and simplifying the resulting inequality, we get the equivalent identity

$$(16)\qquad u_L u_M \alpha_{M,L}(V^M - V^L) + u_L u_H \alpha_{H,L}(V^H - V^L) + u_H u_M \alpha_{H,M}(V^H - V^M) > 0$$

where $\alpha_{S_1,S_2} = \frac{\alpha_{S_1}}{1-\alpha_{S_1}} - \frac{\alpha_{S_2}}{1-\alpha_{S_2}}$, for each state $S_1, S_2 = H, M, L$. Since all of the terms on the left side of (16) are positive, the inequality holds.

(c) Heuristically, this is a result of the monotonicity identity $E^H[X_0] > E^M[X_0] > E^L[X_0]$ (higher the quality type, the larger is the number of good evaluations, on average) and part (b). A rigorous proof is quite lengthy and is omitted

Proof of Lemma 2. Given $V_1 = V_1^S$, the random variable X_1 has a Binomial distribution with parameters N_1 and β_s (with respect to probability measure P). The rest is similar to the proof of Lemma 1 above.

Proof of Lemma 3. (a) The result is clear when the (conditional) expected value of the underwriters' total profit is written as a linear function of l_1. Since $E[V_2|\Omega_1] = V_1$, the expected net wealth has the form

$$l_1 f_1 \Delta_1 \{(c_2 - f_b)V_1 - (c_2(-f_b) - c_1)V_{IPO}\} + \textit{other terms}$$

where $l_1 f_1 = f_b l$.

(b) Writing $\Delta^u = l_0 f_0 \Delta_0 + f_1 \Delta_1 l_1$, and noting that $\Delta_0 = \Delta_0^*$ and the pair $(l_0, l_1) = (l_0^*, l_1^*)$ are known, the owner chooses Δ_1^* based on the expected price levels and the constraint $\epsilon_1 \leq \Delta_0^* + \Delta_1 \leq 1 - l_0 f_0 \Delta_0^* - f_1 \Delta_1 l_1$ from (4). So in the continuation game, the optimal fraction of shares the owner is willing to sell would solve the following maximization problem: Maximize

$$(17)\qquad h(\Delta_1) = \Delta_1 \{V_1(1 - f_1(1-l_1)) - (1 + l_1 f_1)E[V_2(1-f_2)|\Omega_1]\}$$

subject to the condition

$$\epsilon_1 - \Delta_0^* \leq \Delta_1 \leq \frac{1-(1+l_0 f_0)\Delta_0^*}{1+l_1 f_1}.$$

Noting that $E^T[V_2(1-f_2)|\Omega_1] = (1-f_b)E^T[V_2|\Omega_1]$, the result follows from simple linear programming arguments.

Proof of Lemma 4. (a) The proof is similar to that of Lemma 3 after the expected wealth of the underwriter is shown to be in the form

$$l_0 f_0 \Delta_0 \{V_0(1-f_b) - V_{IPO}(1 - c_0 + c_2(1-f_b))\} + \textit{other terms},$$

with $c_0 \frac{v}{V_0}$. Since the "*other terms*" that include Δ_1 are unknown by the underwriters, they can only solve the problem at date 0 using the expression above in this sequential optimization problem.

(b) Now, we have $E^T[V_2(1-f_2)|\Omega_0] = (1-f_b)E^T[V_2]$. The equation (14) is linear in the initial allocations Δ_0 and Δ_1. Moreover, the constraints are linear in Δ_0 and Δ_1. Therefore, linear programming problem applies and the results easily follow.

Proof of Proposition 1. For any given feasible IPO price, underwriter's sub-optimal choice l_0^* as well as the owner's optimal allocations and expected wealth can be obtained as in Lemma 4. Moreover, an optimal IPO price V_{IPO}^* from the owner's perspective always exists in a closed and bounded (compact) feasible set. Given l_0^* and the rule for l_1^*, the owner's best choices result in the optimal expected wealth since the owner has access to complete information to solve her problem. The underwriters' problem can only be optimized sequentially based on the noisy information set and therefore their expected total realized profit is not guaranteed to be optimal under the probability measure based on the perfect information set.

FIGURES

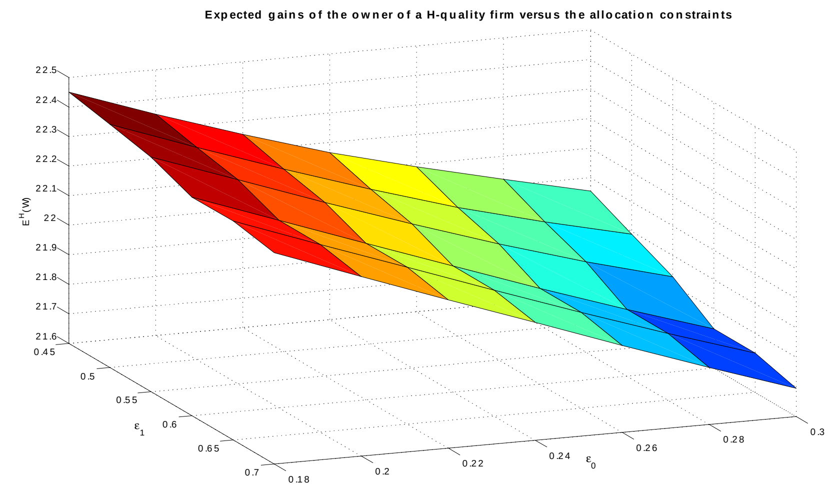

Figure 1. The owner's expected gains decreases with the allocation constraints ε_0 and ε_1

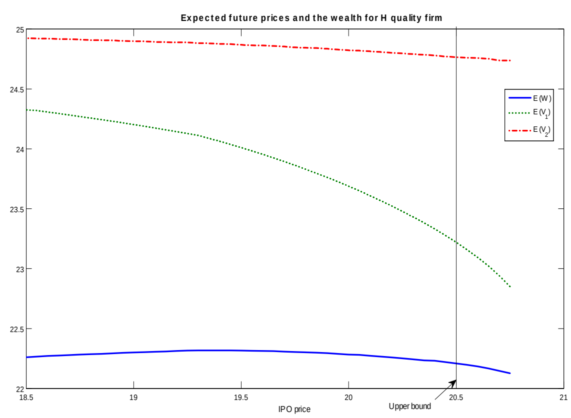

Figure 2. The secondary market price of a high quality firm gradually increases to its actual value as IPO price decreases. Similarly, the price V_2 at date 2 gets much closer to the actual value than the price V_1, on the average.

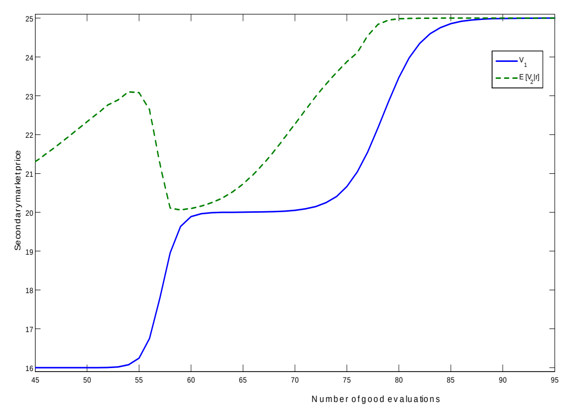

Figure 3. The secondary market price V_1 at date 1 clusters around the actual values $V^L = 16, V^M = 20$ and $V^H = 25$, depending on the number of good evaluations. We also have $E^H[V_2|\Omega_1] \geq V_1$.

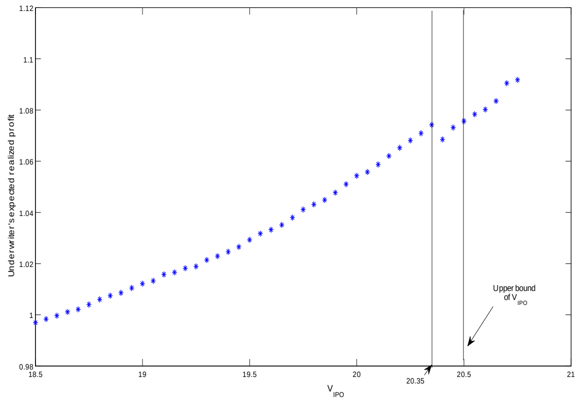

Figure 4. Underwriter's profit from managing the IPO of a high quality firm will be larger at around the upper bound or at the cut-off point for the warrant rates. He would prefer the largest warrant rate if the firm is of a high quality one.

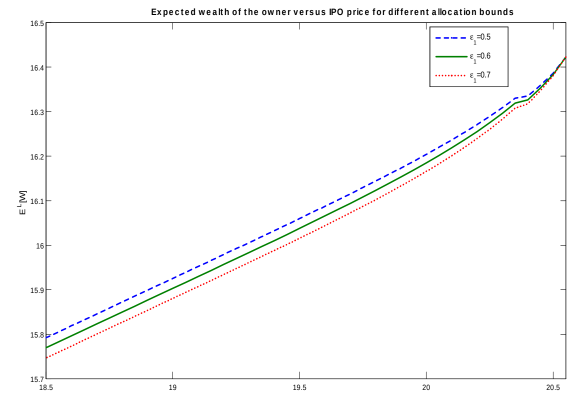

Figure 5. The owner of a low quality firm prefers a larger IPO price, a larger warrant rate, other parameters being constant. Moreover, the expected payoff will be smaller when the firm-specific allocation constraint ε_1 is larger.

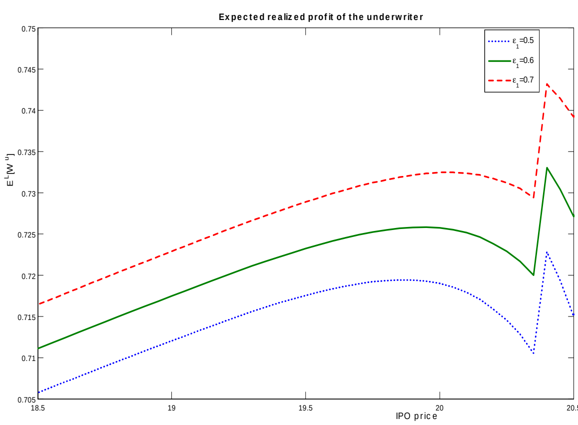

Figure 6. The underwriter's profit is larger either at a point with lower warrant rate (when the IPO price is larger than 20.35 here) or at a much lower IPO price which reduces the IPO related costs.

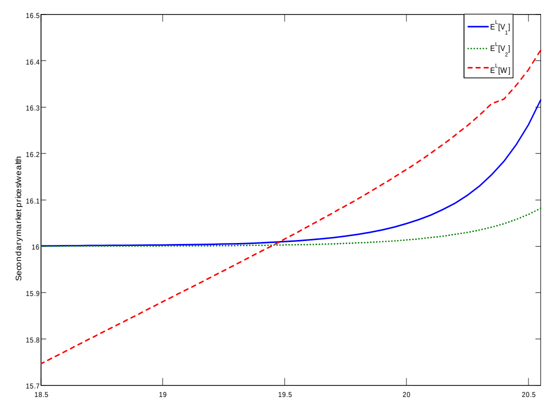

Figure 7. The owner of a low quality firm maximizes the expected payoff at a higher IPO price. The secondary market price will get closer to the actual price by time.

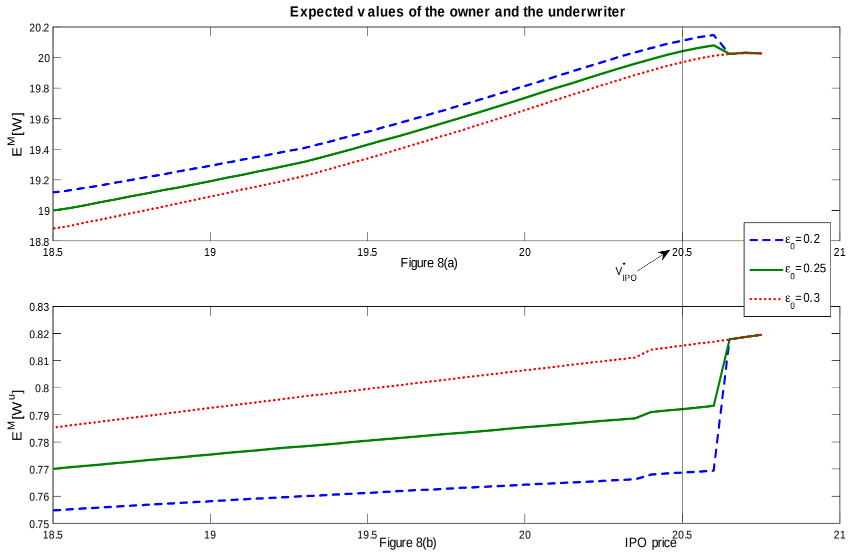

Figure 8. In the case of a medium quality firm, the optimal IPO price will be around the upper bound. The optimal choices for both the owner and the underwriter are quite rich and depends on all the model parameters. There is again a trade-off between both parties regarding the warrant rate but not as extreme as the other (low or high) quality types.

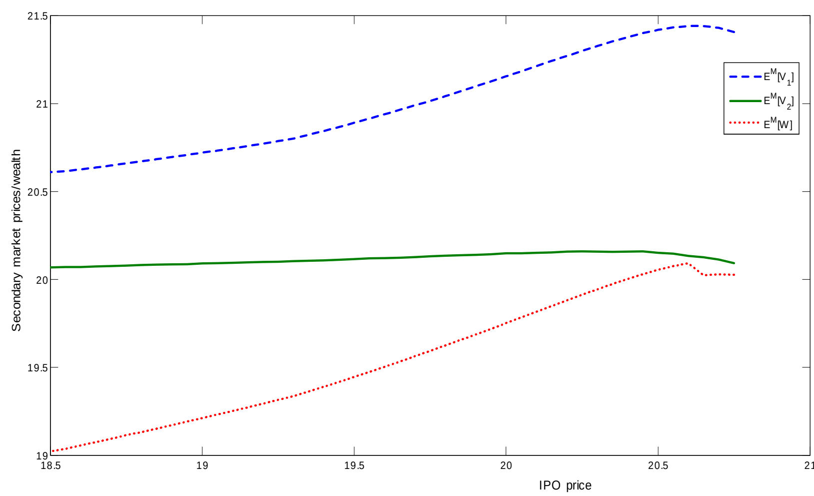

Figure 9. Again, the secondary market prices get closer to the actual value by time although they are not necessarily monotone with the IPO price for a medium value firm. The owner can only afford a small amount of initial price reduction.

ϵ_0	ϵ_1	$\mathbf{V}^*_{IPO}$	$\mathbf{E}[\mathbf{W}(\mathbf{V}^*_{IPO})]$	$\mathbf{E}[\mathbf{W}^u(\mathbf{V}^*_{IPO})]$
0.18	0.5	19.3	22.4226	1.0133
0.2	0.5	19.35	22.3186	1.0229
0.22	0.5	19.45	22.2178	1.0349
0.24	0.5	19.65	22.123	1.0525
0.26	0.5	19.9	22.0352	1.0749
0.28	0.5	20.4	21.959	1.1041
0.3	0.5	20.4	21.8906	1.113

Table I. The equilibrium quantities versus ϵ_0, as ϵ_1 is kept constant.

β_H	V^*_{IPO}	$E[W(V^*_{IPO})]$	$\mathbf{E}[\mathbf{W}^u(\mathbf{V}^*_{IPO})]$
0.80	19.3	22.2829	1.0569
0.85	19.35	22.3186	1.0229
0.90	19.45	22.3445	0.9998
0.95	19.5	22.3631	0.9822

Table II. The equilibrium quantities versus β_H when $\epsilon_0 = 0.2$ and $\epsilon_1 = 0.5$.

REFERENCES

Adams, B., M. B. Slovin, and M. E. Sushka, 2005, "An Analysis of Analyst Coverage of Initial Public Offerings," working paper, University of Portland.

Ali, A. (1997) "Bias in Analysts' earnings forecasts as an explanation for long run underperformance of stocks following equity offerings", Working paper, University of Arizona.

Allen, Franklin, and G. Faulhaber (1989), "Signalling by underpricing in the IPO-market", Journal of Financial Economics, 23, 303-323.

Ausubel, L. and R. Deneckere (1989), "Reputation in bargaining and the durable goods monopoly", Econometrica 57, 511-531.

Barclay, M. and Holderness,C (1989), "Private benefits from control of public corporations", Journal of Financial Economics, 25, 371-395.

Baron, David P. (1982), "A model of the demand for investment banking advising and distribution services for new issues", Journal of Finance, 37, 955-976.

Barry, Christopher B., Chris J. Muscarella, and Micheal R. Vetsuyepens (1991), "Underwriter warrants, underwriter compensation and the costs of going public", Journal of Financial Economics, 29, 113-135

Beatty, Randolph P., and Jay R. Ritter (1986), "Investment banking, reputation, and the underpricing of initial public offerings", Journal of Financial Economics, 15, 213-232.

Beneviste, L.M. and S. Spindt, (1989), "How investment bankers determine the offer price and allocation of new issues", Journal of Financial Economics, 24, 343-361.

Beneviste, L.M., and W.H. Wilhelm (1990), "A comparative analysis of IPO proceeds under alternative regulatory environments", Journal of Financial Economics, 28, 173-208.

Benninga S., M Helmantel, and O. Sarig (2005), "The timing of Initial Public Offerings", Journal of Financial Economics, 75, 115-132.

Bolton, P. and Von Thadden, (1998), “Blocks, Liquidity, and Corporate Control.” The Journal of Finance, 53, 1-26.

Bommel, J.V. (1997), “Messages from the Market to Management: The case of IPO’s”, Working Paper, INSEAD.

Booth, J.R., and R.L.Smith III (1986), “Capital raising, underwriting, and certification hypothesis”, Journal of Financial Economics, 15, 261-281.

Bradley D., B. Jordan and J. Ritter (2003), “The Quiet Period Goes Out with a Bang”, Journal of Finance, 58 (1) 1-36.

—————(2008), “Analysts Behavior Following IPOs: The Bubble Period’s Evidence”, Review of Financial Studies, 21 (1): 101-133.

Brav and Gompers (1997), "Myth or reality? The long-run under-performance of initial public offerings: evidence from venture capital and non-venture capital-backed firms", Journal of Finance 52, 1791–1822.

Busaba, W.Y., L.M. Benveniste, and R.-J. Guo, 2001, "The Option to Withdraw IPOs During the Premarket", Journal of Financial Economics 60, 73-102.

Cantale, S (1997), “The choice of market as a signal of firm quality”, Doctoral dissertation, INSEAD.

Chemmanur, T.J. (1993), “The pricing of initial public offering: A dynamic model with information production”, The Journal of Finance, 48, 285-303.

—, and Fulgeri P. (1996), “A theory of the going public decision”, Unpublished working paper, INSEAD.

——-, and———(1997), “Why include warrants in the new equity issues? A theory of unit IPOs”, Journal of Financial and Quantitative Analysis,forthcoming.

Chen, HC, and J.R. Ritter (2000), “The Seven Percent solution”, Journal of Finance, 55, 1105-1131.

Cornelli, F., and D. Goldreich, 2001, "Bookbuilding and Strategic Allocation", Journal of Finance 56, 2337-2369.

Cornelli, Francesca, and David Goldreich, 2003, "Bookbuilding: How informative is the order book?", Journal of Finance 58, 1415-1443.

Demers and Lewellen (2003), "The marketing role of IPOs: evidence from internet stocks", Journal of Financial Economics, 68, 413-437

Dow, J. and Gary G. (1997), “Stock market efficiency and economic efficiency: Is there a connection?”, Working paper, LBS.

Dugger, M. “Operating Profit Behavior around IPOs in a Rational Equilibrium with Asymmetric Information.” Working Paper. UC Berkeley.

Dunbar, C. (1995), "The use of warrants as underwriter compensation in initial public offerings", Journal of Financial Economics, 38, 59-78

Garfinkel, J.A. (1993), “IPO underpricing, insider selling and subsequent equity offerings: Is underpricing a signal of quality?”, Financial Management, Spring 74-83.

Gomes, A. (1997), “Going public with asymmetric information, agency costs and dynamic trading”, Working Paper, Wharton School, UP.

Grinblatt,J.A. and Hwang C.H.(1989), “Signalling and the pricing of new issues”, The Journal of Finance, 44, 393-421.

Grossman, S. and Stiglitz J. (1980), “On the impossibility of informational efficient market”, American Economic Review, 70, 393-408.

Habib, M., and A. Ljungqvist (1998), “Headline Underpricing and Entrepreneurial Wealth Losses in IPOs: Theory and Evidence,” Working Paper. London Business School.

Holmstrom,B. and Tirole,J. (1993), “Market liquidity and performance monitoring”, Journal of Political Economy, 101, 678-709.

Houston, J. C. James, and J. Korceski (2006) “What a Difference a Month Makes: Stock Analyst Valuations Following Initial Public Offerings” working paper. University of Florida.

Ibbotson, Roger G. (1975), “Price performance of common stock new issues”, Journal of Financial Economics, 2, 235-272.

Ibbotson, R., J. Sinder and J. Ritter (1988), “ Initial public offerings”, Journal of Applied Corporate Finance, 1, 37-45.

Jain, P C (1992), “ Equity issues and changes in expectations of earnings by financial analysts”, Review of Financial Studies, 5, 669-683

James, C. J. Korceski (2006) “Strength of analyst coverage following IPOs” Journal of Financial Economics 82 (2006) 1–34.

Jegadeesh, Weinstein, and I. Welch (1993), "An empirical investigation of IPO returns and subsequent equity offerings", Journal of Financial Economics 34, 153-175

Jenkinson T., A. Ljungqvist (2001), Going Public: The Theory and Evidence on how Companies Raise Equity Finance, Oxford University Press, Second Edition.

Jensen,M.C. and Meckling,W.H. (1976), “Theory of the firm: managerial behavior, agency costs and ownership structure”, Journal of Financial Economics, 34, 305-360.

Keloharju,M. (1993), “The Winner’s curse, legal liability, and the long run price performance of initial public offerings in Finland”, Journal of Financial Economics, 34, 251-277.

————-(1995), “Initial IPO returns and the characteristics of post IPO financing in Finland”, Working paper, Helsinki School of Economics and Business Administration.

Kreps, D. and Wilson R. (1982), “Reputational and imperfect information, Journal of Economic Theory, 27, 253-279.

Leblanc, G. (1997), “Ex-ante moral hazard and optimal Capital structure”, Working Paper, Concordia University.

Leland, H.E. and D.H. Pyle (1977), “Information asymmetries, financial structure and financial intermediation”, The Journal of Finance, 32, 371-387.

Lin, H.W., M. McNichols, and P. O’Brien (2004) “Analyst Impartiality and Investment Banking Relationships” Journal of Accounting Research, 132-156.

Ljungqvist, A. and William J. Wilhelm, 2003, "IPO pricing in the dot-com bubble", Journal of Finance 58, 723-752.

Logue, D., 1973, "Premia on Unseasoned Equity Issues, 1965-69", Journal of Economics and Business 25, 133-141.

Loughran, T. and J.R. Ritter (1995), “The new issue puzzle”, The Journal of Finance, 50, 23-51.

————————(1997), “The operating performance of firms conducting seasoned equity offerings”, The Journal of Finance, 5, 1823-1850

Maug. A. (2001), “Ownership structure and the life cycle of the firm: a theory of the decision to go public.” European Finance Review, 5, 167-355.

Mcdonald, J.G. and A.K. Fisher (1972), “New issue stock behavior”, The Journal of Finance, 27, 97-102.

Megginson, W. L. and K. A. Weiss (1991), ‘Venture capitalist certification in initial public offerings’. Journal of Finance XLVI(3), 879-903.

Michaely, R.,and Shaw,W.H. (1994), “The pricing of initial public offerings: Tests of adverse selection and signalling theories”, Review of Financial Studies, 7, 279-319.

Muscerella, C.J. and Vetsuypens (1989), “A simple model of Baron’s model of IPO underpricing”, Journal of Financial Economics, 24, 125-136.

Pastor L. and P Veronesi, (2003), “Stock Prices and IPO Waves.”, NBER Working Paper no. 9858, July.

Rajan, R. and Servaes, H. (1997), “Analyst following of initial public offerings”, The Journal of Finance, 52.2, 507-529.

Rangan, S. (1998), "Earnings management and the performance of seasoned equity offerings", Journal of Financial Economics, v50, 101-122.

Ritter, J. (1987), “The cost of going public”, Journal of Financial Economics, 19, 269-281.

———(1984), “ The hot issue market of 1980”, Journal of Business, 57, 215-240.

————(1991), “The long run performance of initial public offerings”, The Journal of Finance, 46, 3-27.

Ritter, J. and I. Welch (2002), “ A review of IPO activity, pricing, and allocations.” The Journal of Finance, 57, 1795-1828

Rock, K (1986), “Why new issues are underpriced”, Journal of Financial Economics, 15, 187-212.

Ross, S. (1977), “The determination of financial structure:the incentive signalling approach”, The Bell Journal of Economics, 8, 23-40.

Roell, A (1996), “The decision to go public: an overview”, European Economic Review, 40, 1071-81.

Rydqvist,K. (1997), “IPO-underpricing as tax-efficient compensation”, Journal of Banking and Finance, 21, 295-313.

Sherman and Titman (2002), "Building the IPO order book: Underpricing, and participation limits with costly information", Journal of Financial Economics 65, 3-29.

Smith, C.W. (1977), “Alternative methods for raising capital”, Journal of Financial Economics, 5, 273-307.

Spatt, C. and S. Srivastava (1991), "Preplay communication, participation restrictions, and efficiency in initial public offerings ", Review of Financial Studies, 4:709-726

Subrahmanyam A. and S. Titman. (1999), “The going public decision and the development of financial markets.” Journal of Finance, 54, 1045-1082.

Teo, SH, I. Welch, and TJ Wong (1998) “Earnings management and the underperformance of seasoned equity offerings” Journal of Financial Economics 50 (1998) 63-99

Tinic, Seha M.(1988), “Anatomy of initial public offerings of common stock”, The Journal of Finance, 43, 789-822.

Torstila, Sami (2001), "The distribution of the fees within the IPO syndicate - Statistical data included", Financial Management, Winter 2001

Weiss, H.K.(1993), “The underpricing of initial public offerings and the partial adjustment phenomenon”, Journal of Financial Economics, 34, 231-250.

— and William J.W.Jr (1995), “Evidence on the strategic allocation of initial public offerings”, Journal of Financial Economics, 37, 239-257.

Welch,I.(1989), “Seasoned offerings, imitation costs, and the underpricing of initial public offerings”, The Journal of Finance, 44, 2, 421-449.

Welch, I.(1996), “Equity offerings following IPO: Theory and evidence”, Journal of Corporate Finance, 2, 227-259.