

## 1.8 Exponential Notation and Order of Operations

### Exponential Notation

For any natural number  $n$ ,

$$b^n \text{ means } b \bullet b \bullet b \bullet b \bullet b$$

### Rules for Order of Operations

1. Calculate within the innermost grouping symbols,  $( )$ ,  $[ ]$ ,  $\{ \}$ ,  $| \quad |$ , and above or below fraction bar.
2. Simplify all exponential expressions.
3. Perform all multiplication and division, working from left to right.
4. Perform all addition and subtraction, working from left to right.

Simplify.

1.  $-3^4$       2.  $(-3)^4$       3.  $-5^3$       4.  $(-5)^3$       5.  $10 \bullet 5 + 1 \bullet 1$

6.  $5 \bullet 3^2 - 4^2 \bullet 2$       7.  $3(5-7)^4 \div 4$       8.  $\frac{7^2 - (-1)^5}{3 - 2 \bullet 3^2 + 5}$

9. Evaluate:  $20 \div a \bullet 4$ , for  $a = 5$

Write an equivalent expression without using parentheses.

10.  $-(5x - 2y - 3z)$

Simplify.

11.  $11n - (3n - 7)$       12.  $4m - 9n - 3(2m - n)$

13.  $-8a^2 + 5ab - 12b^2 - 6(2a^2 - 4ab - 10b^2)$

## 2.2 Using the Principles Together

Solve and check.

1.  $3x + 6 = 30$

2.  $5x - 9 = 41$

3.  $12 - 4x = 108$

4.  $-10y - 2y = -48$

5.  $3.4t - 1.2t = -44$

6.  $5y - 10 + y = 7y + 18 - 5y$

Clear fractions or decimals, solve and check.

1.  $\frac{-1}{2} + x = -\frac{5}{6} - \frac{1}{3}$

2.  $1 - \frac{2}{3}y = \frac{9}{5} - \frac{1}{5}y + \frac{3}{5}$

3.  $1.7t + 8 - 1.62t = 0.4t - 0.32 + 8$

4.  $5(2t - 2) = 35$

5.  $6b - (3b + 8) = 16$

6.  $8(2t + 1) = 4(7t + 7)$

7.  $\frac{3}{4}(3t - 6) = 9$

### 2.3 Formulas

1. Surface Area of a Cube. The surface area  $A$  of a cube with side  $s$  is given by:  $A = 6s^2$   
Find the surface area of a cube with sides of 3 in.

Solve each formula for the indicated letter.

1.  $A = bh$ , for  $h$

2.  $I = prt$ , for  $p$

3.  $P = 2l + 2w$ , for  $w$

4.  $A = \frac{1}{2}bh$ , for  $b$

5.  $Q = \frac{p-q}{2}$ , for  $p$

6.  $A = \frac{a+b+c}{3}$ , for  $c$

## 2.4 Application with Percent

Convert to decimal notation.

1. 49%
2. 91.3%

Find percent notation.

1. 0.29
2. 1.39
3. 0.0095

Solve

1. What percent of 150 is 39?
2. 54 is 24% of what number?

3. What is 40% of 2?

Paul takes out a subsidized federal stafford loan for \$2400. After a year, Paul decides to pay off the interest, which is 7% of 2400. How much will he pay?

In a medical study, it was determined that if 800 people kiss someone else who has a cold, only 56 will actually catch the cold. What percent is this?

A baseball player had 13 hits in 25 times at bat. What percent were this?



## 2.6 Solving Inequalities

Graph on the number line

1.  $y < 2$

2.  $-5 \leq x \leq 2$

Solve using the addition principle. Write the answers in the set builder-notation.

1.  $3x - 9 \geq 2x + 11$

2.  $y - \frac{1}{3} > \frac{1}{4}$

3.  $-8n + 12 > 12 - 7n$

Solve using the multiplication principle. Graph and write set-builder notation for the answers.

1.  $8x \geq 32$

2.  $-16x < -64$

3.  $-2x \leq \frac{1}{5}$

Solve using the addition and the multiplication principles.

1.  $5 + 4y < 37$

2.  $5y - 9 \leq 21$

3.  $8 - 2y > 14$

4.  $1.7t + 8 - 1.62t < 0.4t - 0.32 + 8$

5.  $2 > 9 - \frac{x}{5}$

6.  $3(2y - 3) > 21$

7.  $5(t + 3) + 9 > 3(t - 2) + 6$

## 2.7 Solving Applications with Inequalities

Important Words	<u>Translation</u>
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Is at least	
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Is at most	
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Cannot exceed	
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Must exceed	
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Is less than	
--------------	--

Is more than	
--------------	--

Is between	
------------	--

No more than	
--------------	--

No less than	
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1. A number is greater than or equal to 5
2. At least 400,000 people attended the Million Man March.
3. The amount of acid is not exceed 40 liters (L).
4. The women volleyball team can spend at most \$450 for its awards banquet at a local restaurant. If the restaurant charges a \$40 set up fee plus \$16 per person, at most how many can attend?
5. Rod's quiz grades are 73, 75, 89. and 91. What scores on a fifth quiz will make his average quiz grade at least 85?
6. One side of a triagle is 2 cm shorter than the base. What lengths of the base will allow the perimeter to be greater than 19 cm.
7. The width of a rectangle is fixed at 16 yd. For what lengths will the area be at least  $264 \text{ yd}^2$ .

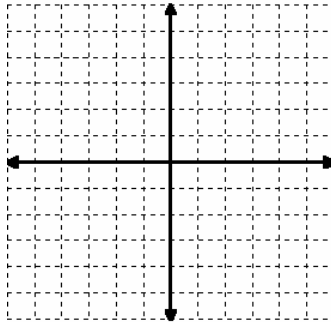
### 3.2 Graphing Linear Equations

To graph a Linear Equation

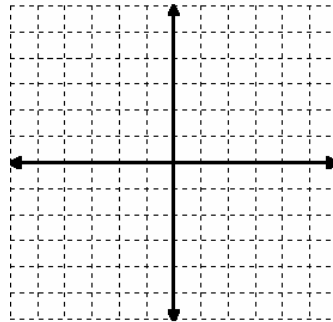
1. Select a value for one coordinate and calculate the corresponding value of the other coordinate. Form an ordered pair. This pair is one solution of the equation.
2. Repeat step (1) to find a second ordered pair. A third ordered pair can be used as check.
3. Plot the ordered pairs and draw a straight line passing through the points. The line represents all solutions of the equation.

Graph each equation.

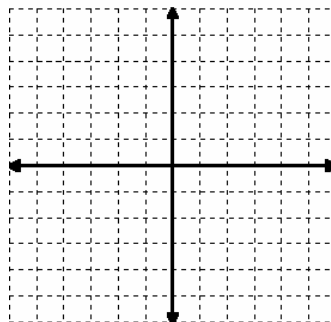
1.  $y=x$



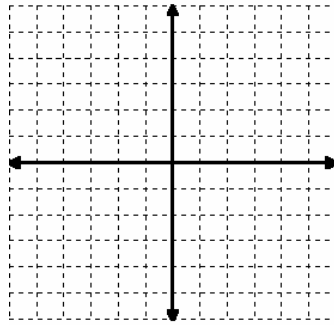
2.  $y=2x - 3$



3.  $y = \frac{5}{2}x + 3$



4.  $x + 2y = -6$



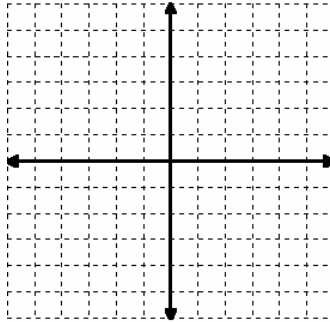
### 3.3 Intercepts. Using Intercepts to Graph. Graphing Horizontal or Vertical Lines

To find y-intercept(s) of an equation's graph, let  $x=0$  and solve for

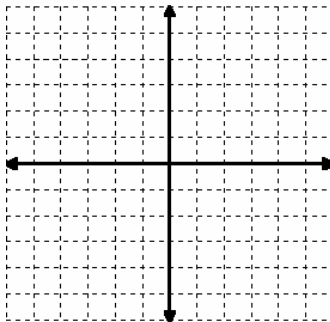
To find x-intercept(s) of an equation's graph, let  $y=0$  and solve for

Find the intercepts. Then graph.

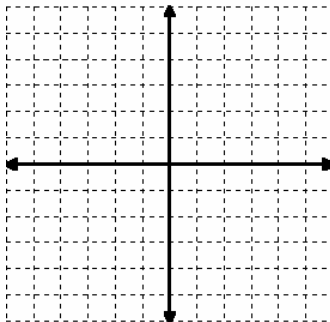
1.  $3x + 2y = 12$



2.  $2x - 3y = 6$

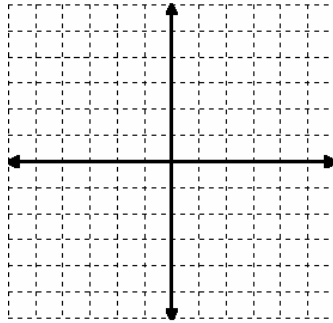


3.  $y = -3 - 3x$

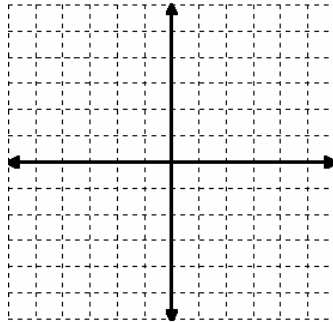


Graphing Horizontal or Vertical Lines

1. Graph  $Y=3$



2.  $X=5$



### 3.4 Rates of Change. Visualizing Rates

A rate is a ratio that indicates how two quantities change with respect to each other.

Car rentals. On February 10, Maggie rented a Chevy Blazer with a full tank of gas and 13,091 mi on the odometer. On February 12, she returned the vehicle with 13,322 mi on the odometer. The rental agency charged \$92 for the rental and needed 14 gal of gas to fill the tank.

a) Find the Blazer's rate of gas consumption, in miles per gallon.

b) Find the average cost of the rental, in dollars per day.

c) Find the rate of travel, in miles per day.

d) Find the rental rate, in cents per mile.

Bicycle Rentals. At 9:00, Blair rented a mountain bike from the Bike Rack. He returned the bicycle at 11:00, after cycling 14 mi. Blair paid \$12 for the rental.

a) Find Blair's average speed, in miles per hour.

b) Find the rental rate, in dollars per hour.

c) Find the rental rate, in dollars per mile.

Four-year-college tuition. The average tuition at a public four-year college was \$2977 in 1995 and \$3489 in 1998 (Source: Statistical Abstract of the United States, 1999, p. 199).

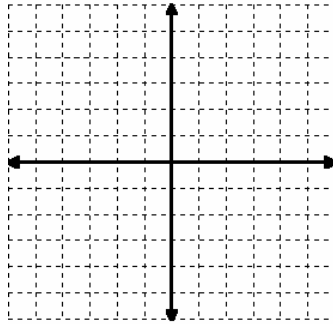
Find the rate at which tuition was increasing.

### 3.5 Rate and Slope. Horizontal and Vertical Lines. Applications

#### Slope

The slope of the line containing points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by

$$M = \frac{\text{change}}{\text{change}} = \frac{\text{rise}}{\text{run}} =$$



Find the slope of the line containing each given pair of points. If the slope is undefined, state this.

1.  $(2, 1)$  and  $(6, 9)$

2.  $(-4, 2)$  and  $(2, -3)$

3.  $(3, 0)$  and  $(6, 2)$

4.  $(0, 9)$  and  $(4, 7)$

Find the slope of each line. If the slope is undefined, state this.

$X = -4$

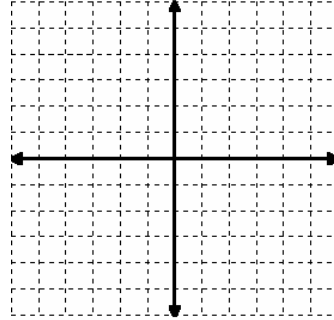
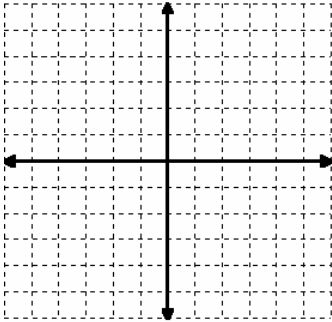
$Y = 17$

### 3.6 Slope-Intercept Form

Using the y-intercept and the slope to graph a line

1. Slope:  $\frac{5}{2}$ , y-intercept (0, 1)

2. Slope:  $-\frac{4}{5}$ , y-intercept (0, 6)



Find the slope and the y-intercept of each line.

1.  $Y = -\frac{3}{8}x + 6$

2.  $-5x + y = 5$

3.  $x - 6y = 9$

4.  $y - 3 = 5$

The Slope-intercept equation

The equation  $y = mx + b$  is called the slope-intercept equation. The equation represents a line of slope  $m$  with y-intercept  $(0, b)$ .

Find the slope-intercept equation for the line with the indicated slope and y-intercept.

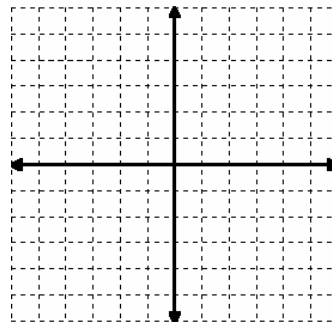
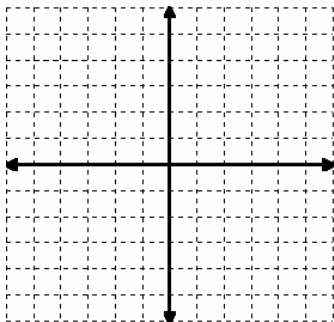
1. Slope:  $-4$ , y-intercept  $(0, 2)$ .

2. Slope:  $\frac{5}{7}$ , y-intercept  $(0, 4)$

Graph

1.  $y = -\frac{3}{5}x - 1$

2.  $3x + y = 2$



If two lines are parallel, then they have the same slope.

If two lines are perpendicular, then the product of their slope is  $-1$ .

Determine whether each pair of equations represents parallel lines.

1.  $y = -3x + 1$   
 $6x + 2y = 8$

Determine whether each pair of equations represents perpendicular lines.

1.  $3y = 5x - 3$   
 $3x + 5y = 10$

### 3.7 The Slope Form

#### The Slope Form of Equation

$Y=mx + b$ ,  $m$  is slope and  $b$  is the  $y$ -intercept.

Write the slope-intercept equation for the line with the given slope that contains the given point.

1.  $m=4$ ; (3, 5)

2.  $m=\frac{4}{5}$ , (7, 1)

3.  $m= -3$ , (-2, -5)

4.  $m=-2$ ; (3, -1)

5.  $m=\frac{3}{2}$ ; (4, 7)

6.  $m=-\frac{3}{4}$ ; (2, 5)

Write the slope-intercept equation for the line containing the given pair of points.

1. (3, 7) and (4, 8)

2. (-2, 0) and (0, 3)

#### 4.1 Exponent and Their Properties

##### The Product Rule

For any number  $a$  and any positive integers  $m$  and  $n$ ,

$$a^m \bullet a^n =$$

Simplify:

1.  $8^4 \bullet 8^3$

2.  $n^3 \bullet n^{20}$

3.  $t^0 \bullet t^{16}$

4.  $(2t)^8 (2t)^{17}$

5.  $(a^8 b^3)(a^4 b)$

6.  $(a^3 b)(ab)^4$

##### The Quotient Rule

For any nonzero number  $a$  and any integers  $m$  and  $n$ ,

$$\frac{a^m}{a^n} =$$

Simplify. Assume that no denominator is zero.

1.  $\frac{4^7}{4^3}$

2.  $\frac{a^{10}}{a^2}$

3.  $\frac{(3m)^9}{(3m)^8}$

4.  $\frac{30n^7}{6n^3}$

5.  $\frac{a^{10} b^{12}}{a^2 b^0}$

##### The Exponent Zero

For any real number  $a$ ,  $a \neq 0$ ,  $a^0 =$

Simplify: 1.  $(8+5)^0$

2.  $(-4)^0 - (-4)^1$

##### The Power Rule

For any number  $a$  and any whole numbers  $m$  and  $n$ ,  $(a^m)^n =$

##### Raising a Product to a Power

For any numbers  $a$  and  $b$  and any whole number  $n$ ,  $(ab)^n =$

##### Raising a Quotient to a Power

For any numbers  $a$  and  $b$ ,  $b \neq 0$ , and any whole number  $n$ ,  $\left(\frac{a}{b}\right)^n =$

Simplify. Assume that no denominator is zero and 0 is not considered.

1.  $(a^3)^8$

2.  $(-3x)^3$

3.  $(xy^4)^9$

4.  $(5x^3)^2(2x^7)$

5.  $\left(\frac{5x}{2}\right)^3$

6.  $\left(\frac{x^5}{y^2}\right)^7$

7.  $\left(\frac{x^3}{y^2z}\right)^5$

8.  $\left(\frac{x^5}{-3y^3}\right)^4$

## 4.2 Polynomials

Terms: A term can be a number, a variable, or a product of numbers and /or variables which may be raised to powers.

A polynomial is a monomial or a sum of monomials.

Monomial:

Binomial:

Trinomial:

Combining Like Terms. Write all answers in descending order.

1.  $5a + 7a^2 + 3a$

2.  $3x^4 - 7x + x^4 - 2x$

3.  $6x^2 + 2x^4 - 2x^2 - x^4 - 4x^2$

4.  $8x^5 - x^4 + 2x^5 + 5x^4 - 4x^4 - x^6$

5.  $\frac{1}{6}x^3 + 3x^2 - \frac{1}{3}x^3 + 7 + x^2 - 10$

6.  $7.4x^3 - 4.9x + 2.9 - 3.5x - 4.3 + 1.9x^3$

Evaluate:  $-2x^3 - 4x^2 + 3x + 1$ , for  $x = -2$

## Circle

**Circumference : C =**

**Area: A =**

Find the circumference of a circle with radius 12 cm.

Find the area of a circle with radius 14 cm.

### 4.3 Addition and Subtraction of Polynomials

Add

1.  $(5x + 1) + (-9x + 4)$

2.  $(x^2 - 5x + 4) + (8x - 9)$

3.  $(9a^2 + 4a - 5) + (6a^2 - 3a - 1)$

4.  $(7 + 4t - 5t^2 + 6t^3) + (2 + t + 6t^2 - 4t^3)$

5.  $\left(\frac{1}{3}x^9 + \frac{1}{5}x^5 - \frac{1}{2}x^2 + 7\right) + \left(-\frac{1}{5}x^9 + \frac{1}{4}x^4 - \frac{3}{5}x^5\right)$

### Opposite of a Polynomial

To find an equivalent polynomial for the opposite, or additive inverse, of a polynomial, change the sign of every term. This is the same as multiplying by the polynomial by  $-1$ .

Simplify

1.  $-(-6x + 5)$

2.  $-(-6a^3 + 2a^2 - 7)$

Subtract

1.  $(5x + 6) - (-2x + 4)$

2.  $(a^2 - 5a + 2) - (3a^2 + 2a - 4)$

3.  $(0.5x^4 - 0.6x^2 + 0.7) - (2.3x^4 + 1.8x - 3.9)$

4.  $(8x^5 + 3x^4 + x - 1) - (8x^5 + 3x^4 - 1)$

#### 4.4 Multiplication of Polynomials

##### To multiply Monomials

To find an equivalent expression for the product of two monomials, multiply the coefficients and then multiply the variables using the product rule for exponents.

Multiply

1.  $(4x^3)7$

2.  $(-x^6)(-x^2)$

3.  $(0.3x^3)(-0.4x^6)$

4.  $\left(-\frac{1}{4}x^4\right)\left(\frac{1}{5}x^8\right)$

5.  $(-4y^5)(6y^2)(-3y^3)$

##### The Product of a Monomial and a Polynomial

To multiply a monomial and a polynomial, multiply each term of the polynomial by the monomial.

Multiply

1.  $2x(4x-6)$

2.  $-2x^3(x^2-1)$

3.  $7t^2(2t+1)$

##### The Product of Two Polynomials

To multiply two polynomials P and Q, select one of the polynomials, say P. Then multiply each term of P by every term of Q and combine like terms.

Multiply

1.  $(x+5)(x+2)$

2.  $(a-4)(a+6)$

3.  $\left(a-\frac{2}{5}\right)\left(a+\frac{5}{2}\right)$

4.  $(5x+3)(5x+3)$

5.  $(x^2+x-7)(x+2)$

6.  $(3t+4)(t^2-5t+1)$

## 4.5 Special Products

### The Product of a Sum and Difference

The product of the sum and difference of the same two terms is the square of the first term minus the square of the second term.

$$(A + B)(A - B) = A^2 - B^2$$

### The Square of a Binomial

$$(A + B)^2 =$$

$$(A - B)^2 =$$

### Multiply

1.  $(x + 1)(x - 1) =$

2.  $(5m - 2)(5m + 2)$

3.  $(3x^4 + 2)(3x^4 - 2)$

4.  $(t^2 - 0.2)(t^2 + 0.2)$

5.  $(t^3 + 4)(t^3 - 4)$

6.  $\left(m - \frac{2}{3}\right)\left(m + \frac{2}{3}\right)$

7.  $(2x - 1)^2$

8.  $(4x^3 + 1)^2$

9.  $\left(t - \frac{1}{5}\right)^2$

## 4.6 Polynomials in Several Variables

### Evaluating Polynomials

1.  $x^2 + 5y^2 - 4xy$ , for  $x=5$  and  $y=-2$

2. Lung capacity. The polynomial  $0.041h - 0.018A - 2.69$  can be used to estimate the lung capacity, in liters, of a female with height  $h$ , in centimeters, and age  $A$ , in years. Find the lung capacity of a 20-year-old woman who is 165 cm tall.

Combine like terms.

1.  $8r + s - 5r - 4s$

2.  $m^3 + 2m^2m - 3m^2 + 3mn^2$

3.  $3x^2 + 6xy + 3y^2 - 5x^2 - 10xy$

4.  $3s^2t + r^2t - 4st^2 + 3st^2 - 7r^2t$

5.  $(2r^3 + 3rs - 5s^2) - (5r^3 + rs + 4r^2)$

Multiply

1.  $(5x + y)(2x - 3y)$

2.  $(ab + 3)(a + 3b)$

3.  $(r + t)^2$

4.  $(p^3 - 5q)(p^3 + 5q)$

5.  $(xy + pq)(xy - pq)$

4.7 Division of Polynomials  
Dividing by a Monomial

Divide and check

1.  $\frac{40x^5 - 16x}{8}$

2.  $\frac{50x^5 - 7x^4 + x^2}{x}$

3.  $(20t^3 - 15t^2 + 30t) \div (3t)$

4.  $(18t^6 - 27t^5 - 3t^3) \div (9t^3)$

5.  $\frac{10x^4 + 15x^3 + 5x}{5x^2}$

Dividing by a binomial

1.  $(x^2 - 6x + 8) \div (x - 4)$

2.  $(t^2 + 8t - 15) \div (t + 4)$

3.  $(3x^2 - 2x - 13) \div (x - 2)$

4.  $(10x^2 + 13x - 3) \div (5x - 1)$

5.  $\frac{8t^3 - 22t^2 - 5t + 12}{4t + 3}$

## 4.8 Negative Exponents and Scientific Notation

### Negative Exponents

For any real number  $a$  that is nonzero and any integer,  $a^{-n} =$

### Factors and Negative Exponents

For any nonzero real numbers  $a$  and  $b$  and any integers  $m$  and  $n$ ,  $\frac{a^{-n}}{a^{-m}} =$

### Reciprocals and Negative Exponents

For any nonzero real numbers  $a$  and  $b$  and any integer  $n$ ,  $\left(\frac{a}{b}\right)^{-n} =$

Express using positive exponents. Then, if possible, simplify.

1.  $2^{-4} =$

2.  $5^{-3}$

3.  $(-3)^4$

4.  $t^{-5}$

5.  $a^{-3}b$

6.  $xy^{-5}$

7.  $\frac{1}{z^{-8}}$

8.  $\frac{1}{a^{-12}}$

9.  $\left(\frac{3}{4}\right)^{-2}$

10.  $\left(\frac{x}{3}\right)^{-4}$

12.  $\left(\frac{r}{v}\right)^{-5}$

13.  $\frac{4a^{-6}}{b^{-5}c^{-7}}$

14.  $(x^3y^{-4}z^{-5})(x^{-4}y^{-2}z^9)$

15.  $\frac{12x^{-6}}{8y^{-10}}$

Express using negative exponents.

1.  $\frac{1}{5^2}$

2.  $\frac{1}{y^2}$

3.  $\frac{1}{m^{12}}$

## 5.1 Introduction to Factoring

### Factoring

To factor a polynomial is to find an equivalent expression that is a product.

Factor. Remember to use the largest common factor and to check by multiplying.

1.  $4x^4 + x^2$

2.  $5x^5 + 10x^3$

3.  $6x^2 + 3x - 15$

4.  $10t^5 - 15t^4 + 9t^3$

5.  $5x^4 - 15x^3 - 25x - 10$

6.  $21r^5t^4 - 14r^4t^6 + 21r^3t^6$

### Factor

1.  $b(b + 5) + 3(b + 5)$

2.  $3z^2(2z + 9) + (2z + 9)$

3.  $x^2(x - 7) - 3(x - 7)$

Factor by grouping, if possible, and check.

1.  $6z^3 + 3z^2 + 2z + 1$

2.  $3a^3 + 2a^2 + 6a + 4$

3.  $10x^3 - 25x^2 + 4x - 10$

4.  $6a^3 - 8a^2 + 9a - 12$

5.  $5x^3 + 4x^2 - 10x - 8$

6.  $x^3 + 7x^2 - 2x - 14$

7.  $3x^3 + 15x^2 - 5x - 25$

## 5.2 Factoring Trinomials of the Type $x^2 + bx + c$

Product:

Sum:

Factor completely. Remember that you can check by multiplying.

1.  $x^2 + 6x + 5$

2.  $a^2 + 11a + 30$

3.  $x^2 - 6x + 9$

4.  $a^2 - 4a - 12$

5.  $d^2 - 7d + 10$

6.  $3y^2 - 9y - 84$

7.  $-x^3 + x^2 + 42x$

8.  $-2x - 99 + x^2$

9.  $-5x^2 - 25x + 120$

10.  $x^5 - x^4 - 2x^3$

11.  $x^2 + 20x + 99$

12.  $3x^3 - 63x^2 - 300x$

13.  $50 + 15x + x^2$

14.  $a^2 - 2ab - 3b^2$

15.  $b^2 + 8bc - 20c^2$

### 5.3 Factoring Trinomials of the Type $ax^2 + bx + c$

To factor  $ax^2 + bx + c$ , using the Grouping Method

1. Factor out the largest common factor, if one exists.
2. Multiply the leading coefficient  $a$  and the constant  $c$ .
3. Find a pair of factors of  $ac$  whose sum is  $b$ .
4. Rewrite the middle term,  $bx$ , as a sum or difference using the factors found in step (3).
5. Factor by grouping.
6. Always check by multiplying.

Factor completely. (Factor out the largest common factor, if one exists)

1.  $4a^2 - 4a - 15$

2.  $15x^2 - 19x - 10$

3.  $2t^2 + 5t + 2$

4.  $28x^2 + 38x - 6$

5.  $49t^2 + 42t + 9$

6.  $24x^2 + 47x - 2$

7.  $16t^2 + 23t + 7$

8.  $-70a^4 + 68a^3 - 16a^2$

9.  $-16x^2 - 32x - 7$

10.  $6x^3 - 4x^2 - 10x$

11.  $-2x^2 + 15 + x$

## 5.4 Factoring Perfect-Square Trinomials and Differences of Squares

### Factoring a Perfect-Square Trinomial

$$a^2 + 2ab + b^2 =$$

$$a^2 - 2ab + b^2 =$$

### Factoring a Difference of Squares

$$a^2 - b^2 =$$

Factor completely. Remember to look first for a common factor and to check by multiplying.

1.  $x^2 - 14x + 49$

2.  $x^2 + 14x + 49$

3.  $5x^2 - 10x + 5$

4.  $25x^2 + 10x + 1$

5.  $120m + 75 + 48m^2$

6.  $x^3 + 24x^2 + 144x$

Factor completely. Remember to look first for a common factor and to check by multiplying. (Factoring a Difference of Squares)

1.  $x^2 - 36$

2.  $-64 + m^2$

3.  $3t^2 - 12$

4.  $98 - 8w^2$

5.  $25x^2 - 4$

6.  $4t^2 - 64$

7.  $24x^2 - 54$

8.  $6p^2 - 6q^2$

9.  $1 - a^4b^4$

## 5.5 Factoring: A General Strategy

### To Factor a Polynomial

1. Always look for the common factor first. If there is one, factor out the largest common factor. Be sure to include in your final answer.

2. Then look at the number of terms.

Two terms: If you have a difference of squares, factor accordingly. Do not try to factor

A sum of squares:  $A^2 + B^2$

Three terms: Determine whether the trinomial is a perfect-square trinomial. If so factor accordingly. If not, try grouping method.

Four Terms: Try factoring by grouping.

3. Always factor completely.

Factor completely.

1.  $5x^2 - 45$

2.  $a^2 + 25 + 10a$

3.  $8t^2 - 18t - 5$

4.  $x^3 + 3x^2 - 4x - 12$

5.  $98t^2 - 18$

6.  $20x^3 - 4x^2 - 72x$

7.  $t^4 + 7t^2 - 3t^3 - 21t$

8.  $4x^4 - 64$

9.  $12n^2 + 24n^3$

10.  $4\Pi r^2 + 2\Pi r$

11.  $n^2 + 2n + np + 2p$

12.  $12 + x^2y^2 + 8xy$

13.  $p^2q^2 + 7pq + 6$

## 5.6 Solving Quadratic Equations by Factoring

### Quadratic Equation

An quadratic equation is an equation equivalent to one of the form:  $ax^2 + bx + c = 0$ , where  $a \neq 0$ .

### The Principle of Zero Products

An equation  $AB=0$  is true if and only if  $A=0$  or  $B=0$ , or both. (A product is 0 if and only if at least one factor is 0.

Solve using the principle of zero products.

1.  $(x + 1)(x + 2) = 0$                       2.  $(x + 9)(x - 3)$                       3.  $t(t + 6) = 0$

4.  $12x(4x + 5) = 0$

Solve by factoring and using the principle of zero products.

1.  $x^2 - 6x + 5 = 0$                       2.  $x^2 - 7x - 18 = 0$                       3.  $x^2 + 8x = 0$

4.  $4x^2 = 25$

5.  $x^2 + 16 = 8x$

6.  $6x^2 - 4x = 10$

7.  $(x + 2)(x - 7) = -18$

8.  $t(t - 5) = 14$

9.  $36m^2 - 9 = 40$

10.  $3x^2 - 2x = 9 - 8x$

## 5.7 Solving Applications

### **The Pythagorean Theorem**

In any right triangle, if  $a$  and  $b$  are the lengths of the legs and  $c$  is the length of the hypotenuse, then

1. A number is 2 less than its square. Find all such numbers.
2. One leg of a right triangle is 2 cm shorter than the other leg. The length of hypotenuse is 10 cm. Find the length of each side.
3. Page Numbers. The product of the page numbers on two facing pages of a book is 210. Find the page numbers.
4. A rectangular table in Arlo's House of Tunes is six times as long as it is wide. If the area of the table is  $24 \text{ ft}^2$ , find the length and the width of the table.
5. The length of a rectangular garden is 4 m greater than the width. The area of the garden is  $96 \text{ m}^2$ . Find the length and the width.
6. The height of a triangle is 3 cm less than the length of the base. If the area of the triangle is  $35 \text{ cm}^2$ , find the height and the length of the base.

## 6.1 Rational Expressions

Definition: A rational expression is a quotient of two polynomials.

List all numbers for which each rational expression is undefined.

1.  $\frac{14}{-5y}$

2.  $\frac{a-4}{a+7}$

3.  $\frac{x^2-9}{4x-12}$

4.  $\frac{p^2-9}{p^2-7p+10}$

Simplify by removing a factor equal to 1.

1.  $\frac{12a^5b^6}{18a^3b}$

2.  $\frac{14x-7}{10x-5}$

3.  $\frac{a^2+5a+6}{a^2-9}$

Simplify, if possible.

1.  $\frac{75a^5}{50a^3}$

2.  $\frac{4x-12}{6x}$

3.  $\frac{3m^2+3m}{6m^2+9m}$

4.  $\frac{a^2-4}{a^2+5a+6}$

5.  $\frac{2t^2-6t+4}{4t^2+12t-16}$

6.  $\frac{a^2-1}{a-1}$

7.  $\frac{6x^2-54}{4x^2-36}$

8.  $\frac{6t+12}{t^2-t-6}$

9.  $\frac{4x^2-4x+1}{6x^2+5x-4}$

## 6.2 Multiplication and Division

### The Product of Two Rational Expressions

To multiply rational expressions, multiply numerators and multiply denominators:

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

### The Quotient of Two Rational Expressions

To divide by a rational expression, multiply by its reciprocal:

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}, \text{ where } B, C, D \neq 0$$

Multiply and, if possible, simplify.

1.  $\frac{10}{t^7} \cdot \frac{3t^2}{25t}$

2.  $\frac{t+2}{t-2} \cdot \frac{t^2-5t+6}{(t+2)^2}$

3.  $\frac{x^2+10x-11}{5x} \cdot \frac{x^3}{x+11}$

4.  $\frac{5v+5}{v-2} \cdot \frac{2v^2-8v+8}{v^2-1}$

5.  $\frac{x^2+5x+4}{x^2-6x+8} \cdot \frac{x^2+5x-14}{x^2+8x+7}$

Divide and, if possible, simplify.

1.  $\frac{4}{9} \div \frac{5}{7}$

2.  $\frac{5}{x} \div \frac{x}{12}$

3.  $\frac{x^5}{y^2} \div \frac{x^2}{y}$

4.  $\frac{a+2}{a-3} \div \frac{a-1}{a+3}$

5.  $\frac{x^2-1}{x} \div \frac{x+1}{x-1}$

6.  $\frac{3x^2-27}{x^2+1} \div \frac{x+3}{x-3}$

7.  $\frac{a^2+5a+4}{a^2-2a+1} \div \frac{a^2+8a+16}{a^2-5a-6}$

### 6.3 Addition, Subtraction, and Least Common Denominators

#### The Sum of Two Rational Expressions

To add when the denominators are the same, add the numerators and keep the common denominator:

$$\frac{A}{B} + \frac{C}{B} = \frac{A+C}{B}, \text{ where } B \neq 0$$

Perform the indicated operation. Simplify, if possible.

$$1. \frac{4}{a^2} + \frac{9}{a^2}$$

$$2. \frac{5}{x+2} + \frac{8}{x+2}$$

$$3. \frac{5+3t}{4t} - \frac{2t+1}{4t}$$

$$4. \frac{a^2}{a+3} - \frac{2a+15}{a+3}$$

$$5. \frac{x-5}{x^2-4x+3} + \frac{2}{x^2-4x+3}$$

$$6. \frac{y^2-7y}{y^2+8y+16} + \frac{6y-20}{y^2+8y+16}$$

$$7. \frac{5-3x}{x^2-2x+1} - \frac{x+1}{x^2-2x+1}$$

$$8. \frac{x-7}{x^2+3x-4} - \frac{2x-3}{x^2+3x-4}$$

## 6.4 Addition and Subtraction with Unlike Denominators

To Add or Subtract Rational Expressions Having Different Denominators

1. Find the LCD.
2. Multiply each rational expression by a form of 1 made up of the factors of the LCD missing from that expression denominator.
3. Add or subtract the numerators, as indicated. Write the sum or difference over the LCD.
4. Simplify, if possible.

Perform the indicated operation. Simplify, if possible.

$$1. \frac{5}{x^2} + \frac{6}{x^2}$$

$$2. \frac{4}{xy^2} + \frac{2}{x^2y}$$

$$3. \frac{x+5}{8} + \frac{x-3}{12}$$

$$4. \frac{2a-1}{3a^2} + \frac{5a+1}{9a}$$

$$5. \frac{2c-d}{c^2d} + \frac{c+d}{cd^2}$$

$$6. \frac{5}{x-1} + \frac{5}{x+1}$$

$$7. \frac{2}{x+5} + \frac{3}{4x}$$

$$8. \frac{5}{x+5} - \frac{3}{x-5}$$

$$9. \frac{4x}{x^2-25} + \frac{x}{x+5}$$

$$10. \frac{2}{x+3} + \frac{4}{(x+3)^2}$$

$$11. \frac{4a}{5a-10} + \frac{3a}{10a-20}$$

$$12. \frac{x}{x^2+2x+1} + \frac{1}{x^2+5x+4}$$

## 6.6 Solving Rational Equations

### To Solve a Rational Equation

1. List any restrictions that exist. Numbers that make a denominator equal 0 cannot possibly be solutions.
2. Clear the equation of fractions by multiplying both sides by the LCM of the denominators.
3. Solve the resulting equation using the addition principle, the multiplication principle, and the principle of zero products, as needed.
4. Check the possible solution(s) in the original equation.

Solve. If no solution exists, state this.

$$1. \frac{4}{5} - \frac{2}{3} = \frac{x}{9}$$

$$2. \frac{1}{8} + \frac{1}{10} = \frac{1}{t}$$

$$3. x + \frac{6}{x} = -7$$

$$4. \frac{4}{t} = \frac{5}{t} - \frac{1}{2}$$

$$5. \frac{3}{4x} + \frac{5}{x} = 1$$

$$6. \frac{5}{x-1} = \frac{3}{x+2}$$

$$7. \frac{x+2}{5} - 1 = \frac{x-2}{4}$$

$$8. \frac{2}{t-9} = \frac{t-7}{t-9}$$

$$9. \frac{x-2}{x-3} = \frac{x-1}{x+1}$$

$$10. \frac{1}{x+3} + \frac{1}{x-3} = \frac{1}{x^2-9}$$

## 6.7 Applications Using Rational Equations and Proportions

Distance = (rate) (time)

1. Carpentry. By checking work records, a carpenter finds that Juanita can build a small shed in 12 hr. Anton can do the same job in 16 hours. How long would it take if they work together?

2. Masonry. By checking work records, a contractor finds that it takes Kenny Dewitt 8 hr to construct a wall of a certain size. It takes Betty Wohnt 6 hr to construct the same wall. How long would it take if they work together?

3. Speed of travel. A loaded Roadway truck is moving 40 mph faster than a New York Railways freight train. In the time that it takes the train to travel 150 mi, the truck travels 350 mi. Find their speeds.

4. Cross-country skiing. Gerard cross-country skis 4 km/h faster than Sally. In the time that it takes Sally to ski 18 km, Gerard skis 24 km. Find their speeds.

5. Geometry. For each pair of similar triangles, find the values of the indicated letter.

6. Photography. Wanda snapped 234 photos over a period of 14 days. At this rate, how many would she take in 42 days.

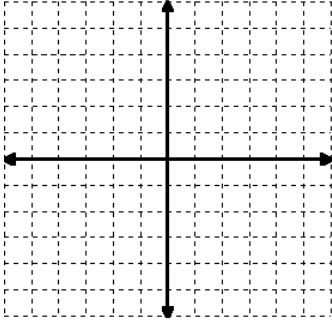
7. Deer population. To determine the number of deer in the Great Gulf Wilderness, a game warden catches 318 deer, tag them, and lets them loose. Later 168 deer are caught; 56 of them have tags. Estimate the number of deer in the preserve.

### 7.1 Systems of Equations and Graphing

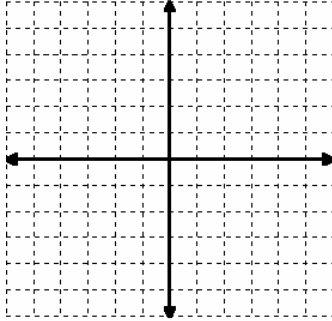
A system of equations is a set of two or more equations that are to be solved simultaneously.

Solve each system of equations by graphing. If there is no solution or an infinite number of solutions, state this.

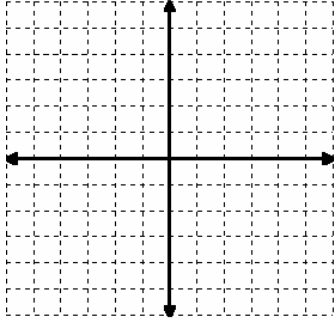
1.  $x - y = 1$   
 $x + y = 3$



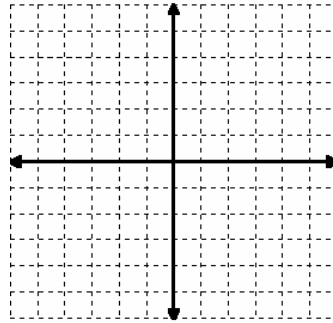
2.  $y = 2x - 5$   
 $x + y = 4$



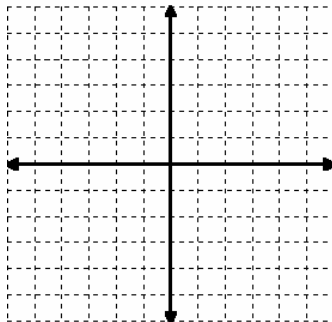
3.  $x = -2$   
 $y = 5$



4.  $x - y = 5$   
 $x - y = 4$



5.  $x + 3y = 6$   
 $2x + 6y = 12$



## 7.2 Systems of Equations and Substitution

Solve each system using the substitution method. If a system has no solution or an infinite number of solutions, state this.

1.  $x + y = 9$   
 $x = y + 1$

2.  $y = x - 3$   
 $3x + y = 5$

3.  $x = y - 8$   
 $3x + 2y = 1$

4. The sum of two numbers is 76. One number is 2 more than the other. Find the numbers.

5. Supplementary angles. Two angles are supplementary. One angle is 8 less than three times the other. Find the measure of each angle.

6. Dimensions of Wyoming. The state of Wyoming is a rectangle with a perimeter of 1280 mi. The width is 90 mi less than the length. Find the length and the width.

### 7.3 System of Equations and Elimination

Solve using elimination method. If a system has no solution or an infinite number of solutions, state this.

1. 
$$\begin{aligned}x + y &= 3 \\x - y &= 7\end{aligned}$$

2. 
$$\begin{aligned}3x - y &= 9 \\2x + y &= 6\end{aligned}$$

3. 
$$\begin{aligned}8x + 3y &= 4 \\-8x - 3y &= -4\end{aligned}$$

4. 
$$\begin{aligned}3x - y &= 8 \\x + 2y &= 5\end{aligned}$$

5. 
$$\begin{aligned}7p + 5q &= 2 \\8p - 9q &= 17\end{aligned}$$

6. 
$$\begin{aligned}5a &= 2b \\2a + 11 &= 2b\end{aligned}$$

7. 
$$\begin{aligned}2p + 5q &= 9 \\3p - 5q &= 4\end{aligned}$$

8. 
$$\begin{aligned}6x - 8 + y &= 0 \\11 &= 3y - 8x\end{aligned}$$

### Applications

1. Local Truck Rentals. U-Haul rents a cargo van for \$19.95 plus 39 cents per mile. Budget rent a cargo van for \$39 plus 30 cents per mile. (Source: Budget rent a car and U-Haul Truck Rentals, July 2000) For what mileage is the cost the same?

2. Complementary angles. Two angles are complementary. Their difference is 26. Find the measure of each angle.

3. Supplementary angles. Two angles are supplementary. One angle measures  $45^\circ$  more than twice the measure of the other. Find the measure of each angle.



## 8.1 Introduction to Square Roots and Radical Expressions.

### Square Root

The number  $c$  is a square root of  $a$  if  $c^2 =$

In general, for any real number  $b$ ,  $\sqrt{b^2} =$

Find the square roots each number.

1. 121

2. 169

Simplify .

1.  $\sqrt{25}$

2.  $\sqrt{361}$

3.  $-\sqrt{441}$

4.  $\sqrt{400}$

Use calculator to approximate each of the following numbers. Round to three decimal places.

1.  $\sqrt{6}$

2.  $\sqrt{43}$

Simplify. Assume that all variables represent nonnegative numbers.

1.  $\sqrt{25a^2}$

2.  $\sqrt{(8ab)^2}$

## 8.2 Multiplying and Simplifying Radical Expressions

### The Product Rule for Square Roots

For any real numbers  $\sqrt{A}$  and  $\sqrt{B}$ , then  $\sqrt{A} \cdot \sqrt{B} =$

### Simplified Form of a Square Root

A radical expression for a square root is simplified when its radicand has no factor other than 1 that is a perfect square.

Multiply.

1.  $\sqrt{7} \cdot \sqrt{5}$

2.  $\sqrt{17} \cdot \sqrt{17}$

3.  $\sqrt{3} \cdot \sqrt{a}$

4.  $\sqrt{3x} \cdot \sqrt{yz}$

Simplify by factoring. Assume that all variables represent nonnegative numbers.

1.  $\sqrt{28}$

2.  $\sqrt{200}$

3.  $\sqrt{4y}$

4.  $\sqrt{49b}$

5.  $\sqrt{125a^2}$

6.  $\sqrt{t^{20}}$

7.  $\sqrt{p^{17}}$

8.  $\sqrt{250y^3}$

9.  $\sqrt{90m^{23}}$

Evaluate  $\sqrt{b^2 - 4ac}$ ,  $a=1$ ,  $b=-3$ ,  $c=-10$

Multiply and, if possible, simplify.

1.  $\sqrt{3} \cdot \sqrt{6}$

2.  $\sqrt{11} \cdot \sqrt{11x}$

3.  $\sqrt{10a} \cdot \sqrt{10a}$

4.  $\sqrt{xy} \cdot \sqrt{xz}$

5.  $\sqrt{x^3y^2} \cdot \sqrt{xy}$

6.  $\sqrt{10xy^2} \cdot \sqrt{5x^2y^3}$

### 8.3 Quotient Involving Square Roots

#### The Quotient Rule for Square Roots

For any real numbers  $\sqrt{A}$  and  $\sqrt{B}$ ,  $B \neq 0$ ,

$$\frac{\sqrt{A}}{\sqrt{B}} = \sqrt{\frac{A}{B}}$$

Simplify. Assume that all variables represent positive numbers.

1.  $\frac{\sqrt{12}}{\sqrt{3}}$

2.  $\frac{\sqrt{72}}{\sqrt{2}}$

3.  $\frac{\sqrt{3}}{\sqrt{48}}$

4.  $\frac{\sqrt{12}}{\sqrt{75}}$

5.  $\frac{\sqrt{48x^3}}{\sqrt{3x}}$

6.  $\frac{\sqrt{20a^8}}{\sqrt{5a^2}}$

7.  $\sqrt{\frac{9}{49}}$

8.  $\sqrt{\frac{100}{49}}$

9.  $-\sqrt{\frac{25}{64}}$

10.  $\sqrt{\frac{7a^5}{28a}}$

11.  $\sqrt{\frac{4x^3}{50x}}$

12.  $\sqrt{\frac{10t^9}{18t^5}}$

#### 8.4 More Operations with Radicals

The sum of like radicals- that is, radical expressions that have a common radical factor- can be simplified.

Add or subtract. Simplify by combining like radical terms, if possible.

1.  $4\sqrt{3} + 8\sqrt{3}$

2.  $9\sqrt{y} + 3\sqrt{y}$

3.  $5\sqrt{6x} + 2\sqrt{6x}$

4.  $12\sqrt{14y} - \sqrt{14y}$

5.  $7\sqrt{2} - 9\sqrt{2} + 4\sqrt{2}$

6.  $5\sqrt{3} + \sqrt{8}$

7.  $\sqrt{25a} - \sqrt{a}$

8.  $2\sqrt{3} - 4\sqrt{75}$

9.  $\sqrt{16a} - 4\sqrt{a} + \sqrt{25a}$

Multiply

1.  $\sqrt{5}(\sqrt{2} + \sqrt{11})$

2.  $\sqrt{6}(\sqrt{15} - \sqrt{7})$

3.  $(\sqrt{5} + \sqrt{11})(3 + \sqrt{11})$

4.  $(1 + \sqrt{5})(1 - \sqrt{5})$

5.  $(8 - \sqrt{7})(3 + 2\sqrt{7})$

## 8.5 Radical Equations

### The Principle of Squaring

If  $a=b$ , then

#### To Solve a Radical Equation

1. Isolate a radical term.
2. Use the principle of squaring (square both sides).
3. Solve the new equation.
4. Check all possible solutions in the original equation.

Solve

1.  $\sqrt{x} = 8$

2.  $\sqrt{x} + 3 = 15$

3.  $\sqrt{2x+4} = 9$

4.  $6 - 2\sqrt{n} = 0$

5.  $\sqrt{4x+7} = \sqrt{2x+13}$

6.  $\sqrt{x} = -5$

7.  $x - 7 = \sqrt{x - 5}$

8.  $x - 5 = \sqrt{15 - 3x}$

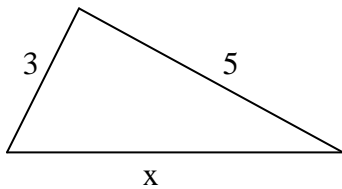
## 8.6 Applications Using Right Triangles

### The Pythagorean Theorem

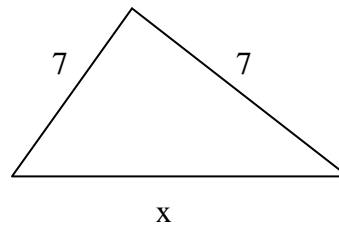
In any right triangle, if  $a$  and  $b$  are the lengths of the legs and  $c$  is the length of the hypotenuse, then

Find the length of the third side of each triangle. Given an exact answer and, where appropriate, an approximation to the nearest thousandth.

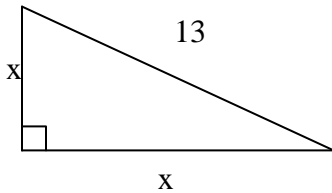
1.



2.



3.



1. Masonry. Find the length of a diagonal of a square tile that has sides 4 cm long.

2. Guy Wires. How long must a guy wire be to reach from the top of a 13-m telephone pole to a point on the ground 9 m from the foot of the pole?

3. Decorating. Celia and Bruce wish to run a string of holiday lights from the top of a 12-ft tall post by their front door to the base of the tree 10 ft away. How long does the string of lights need to be?

## 9.1 Solving Quadratic Equations: The Principle of Square Roots

### The Principle of Square Roots

For any nonnegative real number  $k$ , if  $x^2=k$ , then

Solve. Use the principle of square roots.

1.  $x^2 = 100$

2.  $a^2 = 36$

3.  $n^2 = 13$

4.  $5x^2 = 20$

5.  $25 - 4a^2 = 0$

6.  $4y^2 - 3 = 9$

7.  $(x-2)^2 = 49$

8.  $(a+12)^2 = 81$

9.  $(x-7)^2 = 12$

### 9.3 The Quadratic Formula and Applications

#### The Quadratic Formula

The solutions of  $ax^2 + bx + c = 0$  are given by

Use quadratic formula to solve this equation.

1.  $x^2 + 2x - 2 = 0$

2.  $y^2 + 6y - 2 = 0$

3.  $x^2 - 4x + 4 = 5$

4.  $2x^2 + 3x = 1$

5.  $5t^2 = 100$

Solve using the quadratic formula. Use a calculator to approximate the solutions to the nearest thousandth.

1.  $x^2 + 2x - 2 = 0$

2.  $4x^2 = 4x + 1$

Right triangles. The hypotenuse of a right triangle is 26 yd long. One leg is 14 yd longer than the other. Find the length of the legs.

The length of rectangle is 3 m greater than the width. The area is  $70 \text{ m}^2$ . Find the length and the width.

## 8 Congruent Triangles and Similar Triangles

Two geometric figures are congruent if they have the same shape and size.

**Side-Side-Side (SSS):** If three sides of one triangle are congruent to three sides of a second triangle, the triangles are congruent.

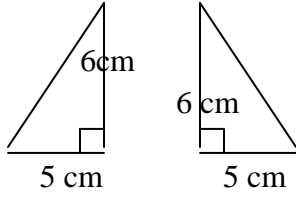
**Side-Angle-Side (SAS):** If two sides and the angle between them in one triangle are congruent, respectively, to two sides and the angle between them in a second triangle, the triangles are congruent.

**Angle-Side-Angle (ASA):** If two angles and the side between them in one triangle are congruent, respectively, to two angles and the side between them in a second triangle, the triangles are congruent.

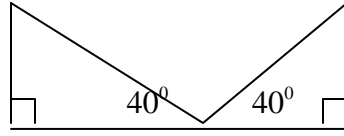
Two triangles are similar if and only if their vertices can be matched so that corresponding angles are congruent and the lengths of corresponding sides are proportional.

Determine whether each pair of triangles is congruent. If they are, tell why.

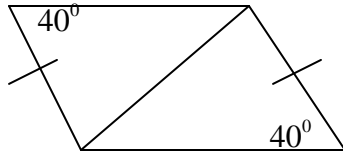
1.



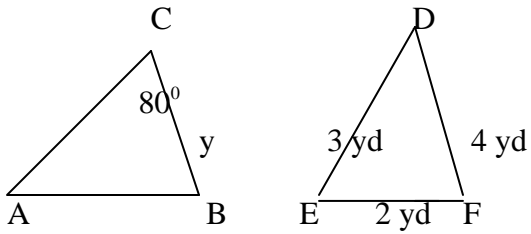
2.



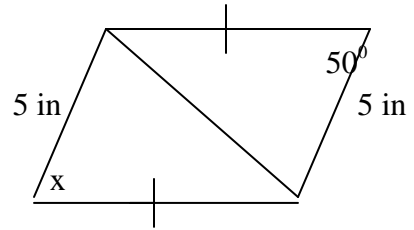
3.



4.  $\triangle ABC \cong \triangle DEF$ . Find  $x$  and  $y$ .

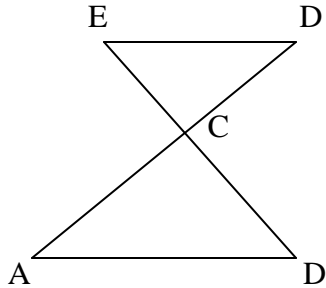


5. Find  $x$ .

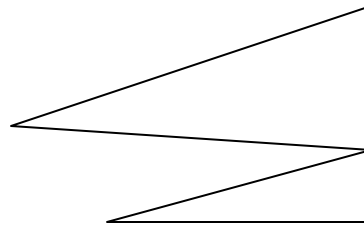


Tell whether the triangles are similar.

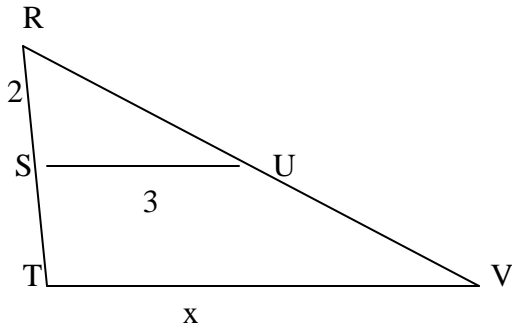
1.



2.



3. If  $SU$  is parallel to  $TV$ ,  $\triangle SRU$  will be similar to  $\triangle TRV$ . Find  $x$ .



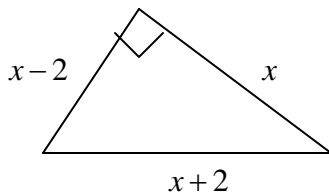
## 9 The Pythagorean Theorem and Special Triangles

Pythagorean Theorem: If  $a$  and  $b$  represent the lengths of two legs of a right triangle and  $c$  represents the length of hypotenuse, then

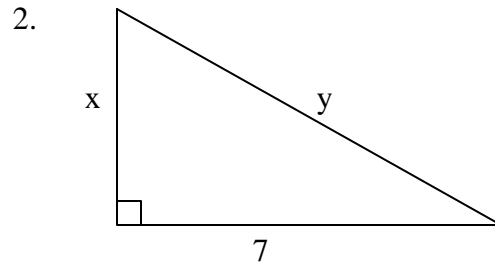
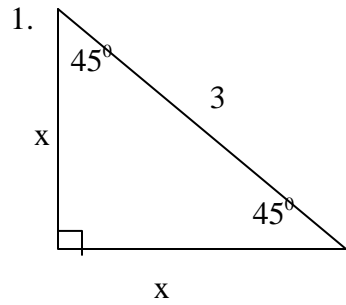
$45^\circ - 45^\circ - 90^\circ$  triangles: An isosceles right triangle is a right triangle with two legs of equal length.

$30^\circ - 60^\circ - 90^\circ$  triangles

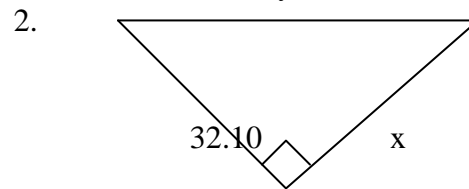
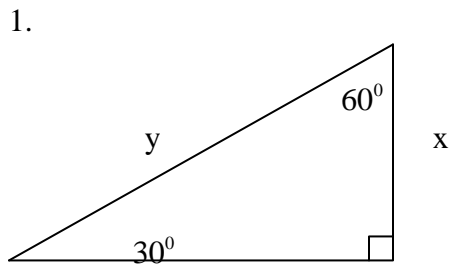
Find  $x$ . Then give the lengths of the other two sides of the right triangle.



Find the missing lengths in each triangle. Give the exact answer and then an approximation to two decimal places when applicable.



Find the missing lengths in each triangle. Give the answer to two decimal places.



## 10 Volume

Definition: The volume of a three dimensional figure is a measure of its capacity. Its unit is cubic unit such as cubic centimeters ( $cm^3$ ).

Cube

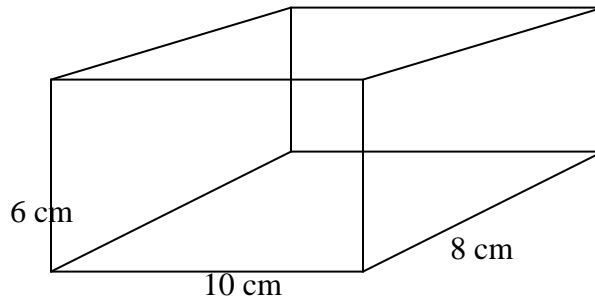
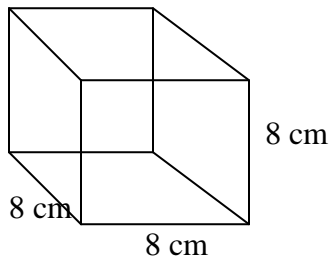
Rectangular solid

Circular cylinder

Find the volume of the following figures. Approximate answer to the nearest hundredth.

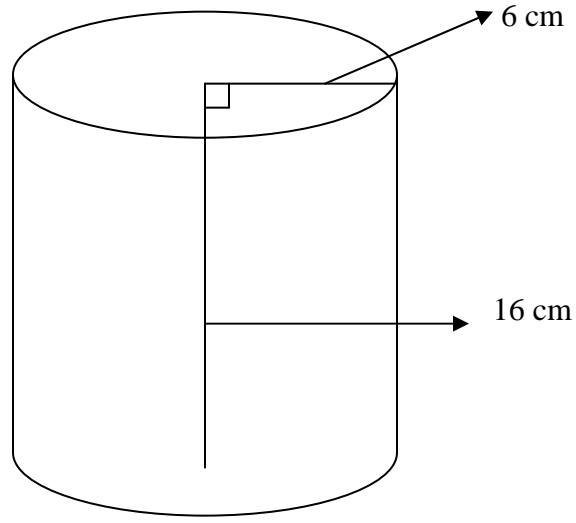
1. Cube

2. Rectangular solid



2.

3. Cylinder



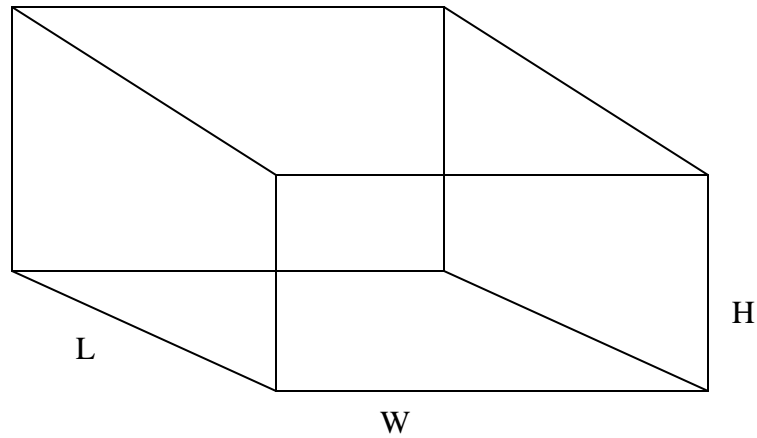
1. The volume of a cube is  $216 \text{ cm}^3$ . What is the length of a side of a cube?

2. The volume of a circular cylinder is  $208\pi \text{ cm}^3$ , and its height is 13 cm. What is the radius of its circular base? Round your answer to the nearest hundredth.

3. The volume of a circular cylinder is  $300\pi \text{ in}^3$ , and the radius of its circular base is 15 in. What is its height?

## 11 Surface Area

The total surface area of a prism is the sum of the area of its bases and its lateral surface area.



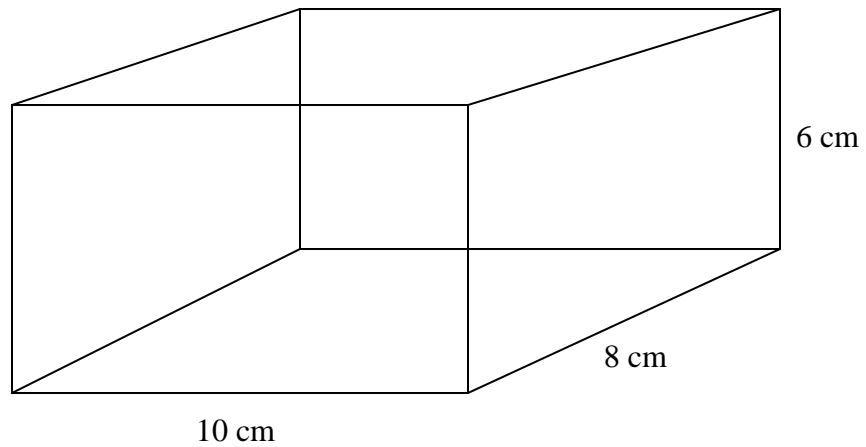
$$\text{Areas of top and bottom} = 2(L \cdot W)$$

$$\text{Areas of front and back} = 2(W \cdot H)$$

$$\text{Areas of left end and right end} = 2(L \cdot H)$$

Find the total surface area of the following figure.

1.



Find the total surface area of the following figure.

2.

