1. Conceptual Understanding

By the end of this course, students will be able to:

1.1. State and interpret key definitions and theorems related to differentiation, integration, and sequences and series of functions.

1.2. Explain the rigorous definitions of concepts such as the Riemann-Stieltjes integral, uniform convergence, and equicontinuity.

1.3. Analyze the differences between pointwise and uniform convergence of sequences and series of functions and their implications.

1.4. Describe the conditions under which a function is differentiable and evaluate the relationships between differentiability and continuity.

1.5. Summarize the key results of the Arzelà-Ascoli theorem and the Stone-Weierstrass theorem, and explain their significance.

2. Proof-Writing and Analytical Skills

By the end of this course, students will be able to:

2.1. Formulate rigorous proofs of theorems involving differentiation, integration, and convergence.

2.2. Construct counterexamples to demonstrate the failure of certain hypotheses in theorems related to series of functions, differentiation, and integration.

2.3. Prove convergence criteria for sequences and series of functions, including applications of the Weierstrass M-test.

3. Higher-Order Thinking and Critical Analysis

By the end of this course, students will be able to:

3.1. Critique the logical structure of proofs presented in class and identify gaps or areas of improvement.

3.2. Compare the Riemann-Stieltjes integral with the standard Riemann integral and justify the need for the generalized definition.

3.3. Synthesize different results from chapters 5, 6 , and 7 to solve advanced problems that span multiple concepts.

3.4. Evaluate the implications of uniform convergence on continuity, integration, and differentiation, and justify their necessity in analysis.

3.5. Develop a deeper understanding of analysis as a foundation for further study in advanced mathematics, including functional analysis and partial differential equations.