Homework 1

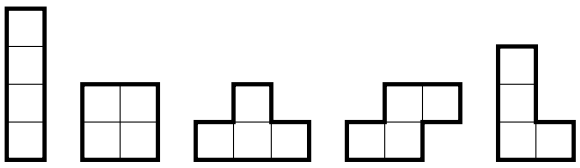
Please show **all** your work on all your assignments. Answers without supporting work will not be given credit.

1. Explain the error in the following "proof" that $2 = 1$.

   Let $x = y$. Then

$$\begin{aligned} x^2 &= xy \\ x^2 - y^2 &= xy - y^2 \\ (x+y)(x-y) &= y(x-y) \\ x + y &= y \\ 2y &= y \\ 2 &= 1. \end{aligned}$$

2. The game *Tetris* is played with five different shapes — the five shapes that can be obtained by piecing together four unit squares:



   For the questions below, we also allow these pieces to be "flipped over." For example,  and  are both allowed. Also, you are allowed to rotate the shapes.

   (a) Is it possible to perfectly cover a 4×5 chessboard using each of these shapes exactly once? Prove that it is impossible, or show by example that it is possible.

   (b) Is it possible to perfectly cover an 8×5 chessboard using each of these shapes exactly twice? Prove that it is impossible, or show by example that it is possible.

3. Prove that at least 2 Sac State undergrads have the exact same 3-letter initials. (Include first, middle and last name in the initials. For instance, Craig Michael Timmons = CMT).

4. Assume that 9 points are chosen from inside the right triangle below and that no three of them form a straight line. Prove that there exist three of these points which form a triangle whose area is less than or equal to 1/2. (Note: the condition that no three from a straight line is simply to guarantee that any three of them can form a triangle.)