Homework 2

1. Prove that if n is odd, then $n^2 + 4n + 9$ is even.

2. Prove that if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.

3. For each of the following pairs of numbers, list all of their common divisors (positive and negative!), and find $\gcd(a, b)$.

   (a) $a = 12$, $b = 330$ (b) $a = -36$, $b = 64$ (c) $a = 7$, $b = -27$

4. Determine the remainder when 3^{302} is divided by 28, and show how you found your answer (without a calculator!). Hint: First figure out $3^3 \pmod{28}$.

5. Assume that $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$. Prove the following.

   (a) $a - c \equiv b - d \pmod{n}$.   (b) $a \cdot c \equiv b \cdot d \pmod{n}$.

6. Prove that for every integer n, either $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$.