Homework 5

1. Look up *Fermat's last theorem* and *Goldbach's conjecture.* Write both down on your homework. Is Fermat's Last Theorem a statement? Is Goldbach's Conjecture a statement?

2. Determine which of the following are statements. Among those that are statements, say whether they are true statements or false statements.

   (a) The sets $\mathbb{Z}$ and $\mathbb{Q}$.
   (b) The sets $\mathbb{Z}$ and $\mathbb{Q}$ both contain $\sqrt{2}$.
   (c) The integer n is a multiple of 5.
   (d) 8675309 is a prime number.

3. Each of the following is either a statement or an open sentence. Express each in the form $P \vee Q$, $P \wedge Q$, or $\sim P$. Make sure you write precisely what your P and Q stand for.

   (a) 27 is both odd and is divisible by 3.
   (b) Either x or y is zero.
   (c) $x \neq y$
   (d) $x \in A \setminus B$

4. Give two examples of an implication ($P \Rightarrow Q$) which is true, but whose converse ($Q \Rightarrow P$) is not true. One example should be a real-world example, while the other should be an example from math involving the integers (and perhaps even numbers, odd numbers, divisibility, sets, or anything else you wish).

5. Without changing their meanings, convert each of the following sentences into a sentence of the form "If P, then Q."

   (a) An integer is even provided it is not odd.
   (b) A geometric series with ratio r diverges whenever $|r| \geq 1$.
   (c) Every polynomial is continuous.

6. Given statements P and Q, write the truth tables for the following.

   (a) $(\sim P \vee \sim Q) \wedge Q$
   (b) $\sim (\sim P \wedge Q)$

7. Determine which of the following are true. If it is true, just say so. If it is false, give a counterexample.

   (a) For all $n \in \mathbb{N}$, we have $(20 - n^2) \in \mathbb{N}$.
   (b) For all $n \in \mathbb{N}$ there exists some $m \in \mathbb{N}$ such that $(m + 1) \mid n$.
   (c) There exists some $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, we have $x^2 = y$.
   (d) For all $x \in \mathbb{R}$ there exists some $y \in \mathbb{R}$ such that $y^2 = x$.
   (e) For all $x \in \mathbb{R}$ there exists some $y \in \mathbb{R}$ such that $y^3 = x$.