

# Homework 7

For each of the below, use a proof by contradiction unless otherwise stated. Also think about whether other proof techniques would work too.

1. Suppose  $a, b \in \mathbb{R}$ .
  - (a) Prove that if  $a$  is rational and  $ab$  is irrational, then  $b$  is irrational.
  - (b) Give an example of two irrational numbers whose product is rational.
  - (c) Give an example of a rational number and an irrational number whose product is rational.
2. Suppose  $m, n \in \mathbb{Z}$ . Prove that if  $4 \mid (m^2 + n^2)$ , then  $m$  and  $n$  are not both odd.
3. Prove that if  $A, B$  and  $C$  are sets,  $A \cap C \subseteq B$ , and  $a \in C$ , then  $a \notin A \setminus B$ .
4. Suppose that  $A$  and  $B$  are sets inside a universal set  $U$ . Prove that  $A^c \cap (B \cap A) = \emptyset$

