Math 108 Due Wednesday April 28th

Homework 8

1. Let $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 7x + 2$. Either prove that f is a bijection or prove that it is not.
2. Let $f : \mathbb{R} \to \mathbb{R}$ where $x^2 - 3x - 2$. Either prove that f is an injection or prove that it is not.
3. Let A and B be finite sets for which $|A| = |B|$, and suppose $f : A \to B$. Prove that f is injective if and only if f is surjective.
4. Let $f : \mathbb{Z} \to \mathbb{Z}$ where $f(x) = x^2 + x$, and let $g : \mathbb{Z} \to \mathbb{Z}$ where $g(x) = 4x + 3$. Determine $(f \circ g)(x)$ and $(g \circ f)(x)$.
5. Give an example of functions f and g such that $f \circ g$ is injective and g is injective, but f is not injective. Write down f, g and $f \circ g$, but you do not need to prove that your example works.
6. Determine whether each of the following is invertible. If it is, write down its inverse (no justification needed). If it is not, then that means it is either not injective, not surjective, or both. If it is not injective, give two distinct values x and y from the function's domain for which $f(x) = f(y)$. If it is not surjective, give an element of the codomain which is not hit by the function. You do not need to prove that your answers are correct. Recall: If the cosine is invertible, its inverse is written "arccos(x)."

   (a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \sqrt[3]{x}$

   (b) $g : \mathbb{R}^+ \to \mathbb{R}$ where $g(x) = x^4$

   (c) $h : [0, \pi] \to [-1, 1]$ where $h(x) = \cos(x)$

   (d) $k : [0, \pi] \to [-2, 2]$ where $k(x) = \cos(x)$