Homework 9 (!)

1. Consider the power set $\mathcal{P}(\mathbb{N})$. Let $\sim$ be the relation $A \sim B$ provided A is a subset of B.

   (a) Determine whether $\sim$ is reflexive, symmetric or transitive. For each, justify your answer.

   (b) Is $\sim$ an equivalence relation? If so, what are its equivalence classes?

2. Consider the relation $\sim$ on the set $\{a, b, c, d\}$ such that this is the complete list of related elements:

$$d \sim d \qquad\qquad b \sim c \qquad\qquad c \sim b$$

$$a \sim a \qquad\qquad b \sim b \qquad\qquad c \sim c$$

   Is $\sim$ reflexive? Symmetric? Transitive? If a property holds, you do not need to justify it. If it doesn't, say why it fails. If all three hold, then $\sim$ is an equivalence relation; in this case, list the equivalence classes.

3. Consider the relation $\sim$ on the set $\{a, b, c\}$ such that this is the complete list of related elements:

$$c \sim c \qquad\qquad c \sim b \qquad\qquad a \sim c$$

$$a \sim b \qquad\qquad b \sim c \qquad\qquad b \sim b$$

   Is $\sim$ reflexive? Symmetric? Transitive? If a property holds, you do not need to justify it. If it doesn't, say why it fails. If all three hold, then $\sim$ is an equivalence relation; in this case, list the equivalence classes.

4. Let $\sim$ be the relation on $\mathbb{Z}$ where $a \sim b$ if $a^2 \equiv b^2 \pmod{4}$. Prove that $\sim$ is an equivalence relation. What are its equivalence classes?