

When and How to Use Math Based Card Tricks in the Classroom

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Abstract

In the paper we compile examples of math-based magic tricks for educators to use in their classroom, and survey research showing that such tricks can help teach and engage students. There is not much overlap between developers of math-based tricks and education researchers—this work helps close that gap. Examples of tricks that have been found in various books and on the internet will be used as examples, and we fully explain the math in the tricks and suggest math course topics when they can be used. We then suggest ways to present these tricks by utilizing education research-based techniques on how to engage students.

When and How to Use Math Based Card Tricks in the Classroom

1 Introduction

Mathematics is a crucial and central part of every student's education. Students begin working with numbers as early as preschool, and are required to take math courses until at least their third year of high school. Moreover, most baccalaureate degrees require at least one math class. And yet, teachers report that their students' competency is lower than their counterparts from past years (Wilson, 2008). Students also report that college math classes are particularly difficult because of the abrupt change in teaching style and time spent in lecture, as compared to high school (Salvaterra, Lare, Gnall, & Adams, 1999). With teachers and students reporting both frustrations and decreases in students' math competency, there is a need to improve students' understanding of math. Previous research on students' math competency discuss various factors, but three common factors are: self-efficacy, attitudes towards math, and math anxiety (Kurbanoglu & Takunyaci, 2012). These three contributing factors can be mistaken as the same, but they are not; each factor is studying a different attribute of a student's perspective on math.

Self-efficacy is a person's belief in their own ability to successfully complete a task (Nicolaidou & Philippou, 2003), a contributing factor in achievement levels in mathematics. Observing self-efficacy is different from observing attitudes towards math and math anxiety because it is the observation of how students perceive their own ability in math. A student may not necessarily believe a subject is hard, but the student could believe that they are unable to grasp a concept. This may lead the student to be unsuccessful in the subject. Self-efficacy is commonly misunderstood as attitude, however, self-efficacy is the perceived ability and how well a student thinks they can do

personally. Contrarily, attitudes are an emotional response to math.

Attitudes are a learned inclination or habit to respond positively or negatively to an object. This is an evaluative response (Mohamed & Waheed, 2011). In previous studies, attitude towards math were tested through the use of the Fennema-Sherman Mathematics attitude scale (Mohamed & Waheed, 2011). Because students' attitudes are impacted by their confidence and the perceived usefulness of math, the scale tested for those two factors. The mean of the test was 3.64, which on that scale was ranked as medium attitude. The students did not have a low average or extremely high average for their attitudes towards math, showing there is still room for improvement. Studies have shown that when students are engaged and interested in their learning, they produce better testing results (Ma & Kishor, 1997). Students are impacted by various factors within the school, such as teachers, as well as outside the school, such as their family or friends. A teacher can impact a student's attitude regarding math in various ways, such as reinforcing stereotypes in the classroom or being uncomfortable with the subject matter themselves. A teacher's lack of confidence in their math abilities will reflect in their teaching (Mooney, Briggs, Hansen, McCullouch, & Fletcher, 2014). Students may also be impacted by sociocultural factors that change their attitudes towards math (Brand, Glasson, & Green, 2006). The main way one's attitude towards math is studied is by observing their emotional response to math.

Indeed, math anxiety is also a significant contributing factor to students' achievement in mathematics. Math anxiety can symptomatically present itself differently for different people. It can be as simple as getting clammy hands when observing anything that the person relates to math. It can also be as extreme as having a panic attack when faced with math, inside and outside the standard classroom setting. Math anxiety incorporates attitudes towards math, but it is different from attitudes towards math and self-efficacy because there is a physiological and psychological effect.

Recreational math is the use of puzzles, games, riddles, and other creative recreational activities that are based in mathematical principles. It has been shown that there are many positive outcomes on students' interest and understanding when a teacher uses recreational math in the classroom in the form of educational games (Schacter, 1999). Studies that have looked at recreational math in classrooms assess the math concepts and recreational math techniques that are being used (Lindström, Gulz, Haake, & Sjöden, 2011), (Abdul Hadi, Mohd Noor, Abd Halim, Alwadood, & Khairol Azmi, 2013), (Pareto, Schwartz, & Svensson, 2009).

One version of recreational math that has been suggested is magic tricks which work based on mathematical principles; sometimes called "Mathematical Card Magic" (Mulcahy, 2013) and "Magical Mathematics" (Diaconis & Graham, 2012). Math and logic have been used for centuries in magic tricks, and the formal development and popularization of these tricks can be traced back at least as far as 1957, when Martin Gardner began writing a monthly column called "Mathematical Games" in *Scientific American* (Berlekamp & Rodgers, 1999). Martin Gardner, along with other "mathemagicians," have been able to combine principles from mathematics with the excitement of magic to produce mathemagical tricks. Despite their promise, it has not been noted if math teachers are aware of magical mathematics as a means to help students understand mathematical concepts. Let alone has it been noted if educators have in-depth instructions on how to use math magic to maintain the excitement of math, and engage their students.

To teach math effectively there have been various studies into what affects math education, such as math anxiety and attitudes towards math. These studies show that changing an instructor's teaching strategies can improve their students' education. Previous studies on recreational math have primarily tested the application of recreational math in the classroom, but do not study the educator or facilitators and vice versa; the studies that have been done in math education lack the use of considering how to facilitate recreational math techniques and focus solely on studying the educator or student.

This study aims to be another step in that direction. There is a combination of math-based card tricks with strategies found in math education research in order to utilize math magic in the classroom and improve learning. This study will discuss magic tricks and the mathematical explanations behind the tricks, along with how an educator can engage students and make the connection between the math and the magic.

2 Literature Review

2.1 Attitudes Towards Mathematics, Math Anxiety, and Self-efficacy

Self-efficacy has been shown to influence students' mathematical achievement (Ayotola & Adedeji, 2009). Acknowledging a student's self-efficacy would involve asking how capable they believe they are at specific concepts rather than how they feel about math. Math is taught to students at a very young age, and continues well into their teenage years or their twenties, so their attitudes and self-efficacy have time to evolve. Young students have been found to be overconfident in their mathematical abilities (Erickson & Heit, 2015). Overconfidence has been found to give the student motivation and drive to continue with math. (Erickson & Heit, 2015) found that the problem is in protecting students from disappointment; as students get older, continuous disappointment with math can alter self-efficacy and create a longstanding belief of incapability.

There is an association between a student's perspective of an academic subject and their academic success within that subject (Vandecandelaere, Speybroeck, Vanlaar, De Fraine, & Van Damme, 2012). Understanding how a student perceives their environment plays a key role in understanding that student's academic success and their attitudes towards a subject. The mathematics attitude is composed of three areas:

1. Self-confidence in correlation with the perceived difficulty level of learning math,
2. The student's enjoyment with the subject, and
3. Their beliefs as to the usefulness of math and its relevance in their lives (Vandecandelaere et al., 2012).

If students do not receive a good foundation or develop an interest as a child, there is a chance that the student will have difficulty viewing math positively. Research shows that positive attitudes towards math progresses into success in math (Ma & Xu, 2004). Attitudes towards math are influenced by various factors from inside the classroom and personal experience (Brand et al., 2006). Teaching materials, classroom organization, the teacher's nature and traits, learning connections to real-world examples, and peers' attitudes toward the subject are all factors that can influence a student's attitude from within the classroom (Mohamed & Waheed, 2011)). All these factors can contribute to how a student reacts to math.

It has been noted that students often label mathematics as a subject that is cold, difficult, abstract, and theoretical (Mohamed & Waheed, 2011). Math negativity can have consequences for students learning in the classroom. Some of the consequences that can be observed are: stress, behavior problems, avoidance of advanced math courses, low motivation, less participation and boredom (Willis, 2010). Furthermore, a negative attitude towards math can lead to negative consequences in their academics even outside of their math classes (Perry, 2004).

Math anxiety is similarly detrimental. Math anxiety is not just the negative emotion one may have about math; but a force powerful enough to send people into a state of distress any time they are in contact with numbers. Unlike attitudes towards math, math anxiety affects a student's physiological and psychological welfare. Maloney and Beilock (2012) state that math anxiety is the "adverse emotional reaction to math or to the prospect of doing math." (Maloney & Beilock, 2012). Examples beyond the classroom are numerous, but they include such routine things as reading a

cash register receipt. Rancer et al. (2013) did research into the pedagogy and a student's perspective of a research methods course. The research shows that most students found the statistical concepts challenging in the class (Rancer, Durbin, & Lin, 2013). The higher a student's math anxiety with a topic, the higher their perceived difficulty with the topic. These same students also had lower levels of their perceived understanding of topics. These findings also show that students who exhibit math anxiety do not only struggle in math courses, it can affect them outside of math classrooms.

2.2 Alleviating Math Anxiety

To coincide with looking at how math anxiety and negative attitudes towards math affect a student's learning, it is also important to look into the research about correcting math anxiety and attitudes. Looking at ways to correct math anxiety will help educators better understand how to make math classes workable for more students, not just those who live without math anxiety.

One approach to helping mathematically anxious students is to focus on when the anxiety begins. Math anxiety was initially believed to have begun in the later years of a student's education, usually appearing around the start of junior high when the course material gets more difficult (Ashcraft & Krause, 2007). Recent research, though, suggests that math anxiety can begin for some students as young as in the first grade (Ramirez, Gunderson, Levine, & Beilock, 2013). Math anxiety can be developed from a classroom setting where the teachers display signs of being anxious about their own math abilities (Maloney & Beilock, 2012). Students can also gain math anxiety from a teacher who endorses the stereotype that math is for boys and reading is for the girls (Maloney & Beilock, 2012). Teachers must be made aware of their own predispositions to effectively create an environment that does not promote the development of math anxiety.

Students have been noted saying math is abstract and from their experience in education, teachers have failed to make connections to the world outside the classroom (Perry, 2004). To fa-

cilitate math and alleviate some frustrations, teachers should encourage communication in the class through conversing, problem solving, and writing (Furner & Berman, 2003). The National Council of Teachers of Mathematics (Council et al., 1997) have suggested educators use manipulatives (physical objects for hands-on learning) and encourage realistic problem solving exercises. Zemelman et al. (1998) found that some of the best outcomes for students are when educators endorse manipulatives, group work, discussion, questions and inferences, explaining an individual's thinking, writing, problem solving, and the use of technology.

Helping students maintain a positive attitude towards math will help them cope with their math anxiety and attitudes. Some suggestions from NCTM (1989, 1995b) (Council et al., 1997) are:

1. Be willing to help different learning styles
2. Design positive experiences in math class
3. Emphasize that everyone makes mistakes
4. Make math relevant
5. Permit various approaches
6. Stress originality and quality thinking rather than manipulation of formulas
7. Characterize math as a human endeavor

Some of these techniques were observed in the Programme for International Student Assessment (PISA) in 2012 as discussed about by Vashchyshyn and Chernoff (2016) . PISA produced data on the mathematical, reading and scientific literacy of a sample of fifteen-year-olds from each participating country (Vashchyshyn and Chernoff, 2016). In that year, Quebec had some of the top performing schools in mathematics, which the study concluded was made possible through some of their particular efforts. Quebec has multiple programs to promote math and logic among students,

and the programs are not only aimed at top students, but all students (Vashchyshyn & Chernoff, 2016). They also emphasize teachers' pedagogical understanding of mathematics. This is an important piece of a student's education because the more their teachers know the more likely they are to receive a great education. For teachers to be able to effectively improve their students' enjoyment of the subject, learning strategies ought to raise their students' confidence, which in return will have a positive association with their comprehension of the subject. To increase a student's mathematical knowledge, the student should be comfortable in their environment, gain confidence, and have an interest in the subject (Vashchyshyn & Chernoff, 2016; Mayer, Fennell, Farmer, & Campbell, 2004).

2.3 Engaging Students with Recreational Math and its Effects

Studies have been conducted on the correlation between student's math anxiety and the use of a math game (Mohd Nordin, Md Tahir, Kamis, & Khairul Azmi, 2013; Abdul Hadi et al., 2013). A program called 'Mini Hari Matematik' was implemented with a group of 4th and 5th grade students, ages ten to eleven. The students were given the exact same pre-test and post-test survey after playing a computer game. The results showed that a student's confidence increased by 0.03% (Mohd Nordin et al., 2013). The study altogether tested confidence, anxiety, motivation, and awareness. Confidence and anxiety had the greatest positive correlation of 0.65 (Mohd Nordin et al., 2013). Students' perceptions of what they were learning had changed because they were comfortable, which allowed for students to begin feeling confident.

Some may question whether math games have an impact on what students are learning specifically. Forty 8-10 year-old students played a game that was designed to help them understand the base 10 concepts in numbers (Lindström et al., 2011). For example, the '2' in '20' stands for two sets of 10. The study utilized an educational game by Pareto et al.'s (Pareto et al., 2009), which was not given a formal name. The pedagogical principles behind the study were to test four areas

1. Numerical representations help develop understanding
2. Guiding students' attention supports mathematical concepts and principles
3. Teaching to others enforces reflection
4. Collaboration between students helps develop understanding (Lindström et al., 2011).

Various techniques were implemented in making the game, such as using colors and graphics to keep the students' attention, and conversational questions to assess the students' reasoning (Lindström et al., 2011). Overall the game was seen to positively work well with the four pedagogical principles being tested for. The researchers did find some mismatches in the game to the pedagogical practices, these were areas where the designer intended to enforce pedagogy and instead it did the opposite. The mismatches stemmed from students not being able to comprehend some of the graphics and instructions. Students also perceived some of the conversational questions as a form of punishment when they got a question wrong (Lindström et al., 2011). These mismatches left room for improvement between using math games and technology in the classrooms, and how to run such a study.

Dragonbox 12+ is an android application designed for students to better their basic algebra skills such as addition, subtraction, expansion, operations of variable and substitution (Siew, Geoffrey, & Lee, 2016). It is a game application that was tested on 60 eight grade students (age 14). This study tested a student's algebraic thinking and attitudes, rather than just the success rate of the students because they are in an engaging environment. Algebraic thinking in relation to the research is "abstract letter representation and manipulation and application of algebraic expressions to perform procedures in non-routine and routine problems" (Siew et al., 2016). Students were separated into two groups: the control group and the experimental group. Each group contained 30 students, which both participated in a pre-and post-test quasi experiment. The group of students in the experimental group, used Dragonbox 12+ while learning. The control group was taught through

imitation and repetition. The students in the experimental group returned positive results for their level of confidence and usefulness of the math. This study shows that Dragonbox 12 + improved the algebraic thinking and attitudes towards math in the students.

Dragonbox 12 + and Mini Hari Matematik are just two of many games introduced to students to influence their learning. While this research does not use math games or applications, it will be using common methods all with the same goal—to increase a student’s comprehension of math.

2.4 Magic Tricks in the Classroom

Little research has been done in the terms of directly using mathemagic in the classroom. Existing research provides suggestions on how to go about using mathemagic in the classroom and strategies to make the trick go smoothly, such as in Deviney (2010). In this study, Deviney gives four principles on how to use magic in the classroom:

1. Do not let the illusion overpower the lesson,
2. Practice before you present,
3. Never repeat a trick to the same audience more than once, and
4. Do not explain the trick because it takes away from the excitement and mystery

These are good suggestions if the purpose of the magic trick is just to engage students in the lecture or the instructor. Our goals are to use the tricks to teach and illustrate mathematical concepts, and to stimulate discussion and thought into the mathematical concepts being taught. In order to attain these goals, one must use repetition and explanation.

Many mathemagic tricks can be found in books and on the internet. However the math

is sometimes explained, and the trick is left up to reader to learn or vice versa. Other times the trick is demonstrated and the math is noted, but a complete explanation of the math is omitted. Matthews (2008) gives tricks for teachers to implement in their classrooms. The tricks provided are based on an array of skills that students learn in the K-12 system such as addition and algebra. The article suggests tricks to use in the classroom, but does not study the effects of the magic in the class (Matthews, 2008). There are also books, such as *Magical Mathematics* by Diaconis and Graham (2012) that give a plethora of math magic tricks along with a thorough explanation of the principles behind them (Diaconis & Graham, 2012). The tricks discussed in the methodology portion of this paper will be replicas of tricks used in these texts.

3 Examples- Mathemagical Tricks

3.1 Overview

The tricks in this paper are all based on mathematical principles; there are none from pure deception or trickery. Over the last century many tricks have been developed, and in this research we highlight a few of these tricks. Educators are advised to practice a trick many times before performing it to a class; as a well-rehearsed trick is not only more entertaining but also adds more mystery to why it works. This in turn better engages students, facilitates learning, and promotes discussion.

When reading the tricks, the presenter is referred to as the “magician” and the audience member is referred to as the “participant.”

3.2 Trick 1: Counting Kings

The magician will need a standard deck of 52 cards for this trick. Begin by having someone shuffle the deck as many times as they please; doing this gives a sense of legitimacy to the trick. Retrieve the deck of cards. While holding the cards face down in your hand, flip over the top card and place it on the table, and then “count up to the king.” For example, if your first card was a 4 you would place another card on top of the 4 as you say “five,” then place another on top as you say “six,” and so on, placing one on top as you say “eight,” “nine,” “ten,” “jack,” “queen” and “king.” After you are done counting to the king, flip the pile of cards over so that they are face down, and set this stack aside. Turn over the next top card in the deck and again count to the king. Continue doing this, making as many piles as possible. When you have reached a point that you do not have enough cards left to count to a king, discard the remaining cards. In the case that you do not have a discard pile, the trick will still work.

Now, turn around while you let a participant mix around the stacks, so you can not remember which stacks are where. Then have a participant choose three of these piles, and discard the rest. Have the participant then turn over the top card in two of the piles and place them back on top of the stack that it was chosen from, face up. Your goal is to guess the top card in the third pile.

First let the audience members and participant look at the top card then place it back on top of the pile, face down. Now the magician, should count the number of cards in the discard pile. The magician takes this number, subtracts ten, and then subtracts the ranks of the two face-up cards. This is the special number. The magician announces to the audience that she can guess the rank of the third card, and states the special number.

– Why it works –

Notice that in the example above, on top of the 4 we placed 9 cards while counting to the king. If the top card was an 8 we would have placed 5 cards on top (for 9, 10, jack, queen, king). In the first example if you take 13 and subtract off the card's rank of 4, you get 9 which is the number of cards we placed on top; that is, $13 - 4 = 9$. This property with 13 was also true in the second example: $13 - 8 = 5$. Indeed, in general if the card's value is x , then $13 - x$ cards will be placed on top of x in order to count to the king.

Suppose that the magician is at the point of the trick where there are three piles remaining, you can see two of the top cards, and you counted the cards in the discard pile. Let's say the two cards you can see have ranks x and y , and there are d cards in the discard pile. What you want to find is z , the rank of the card on top of the third pile.

The total number of cards in the deck is 52. Let's now count up the cards by thinking about each pile at a time. The pile with an x on top has this card plus $13 - x$ more cards; so in total there are $14 - x$ cards in this stack. Likewise the stack with a y has $14 - y$ cards and the stack with z has $14 - z$. (Note that we don't know what z is right now, but still that stack has $14 - z$ cards.) Finally, the discard pile has d cards in it. Since these totals must add to 52, we can say that

$$52 = (14 - x) + (14 - y) + (14 - z) + d$$

Solving for z ,

$$52 = 42 - x - y - z + d$$

$$z = d - 10 - x - y.$$

So you see, if you take the number of cards in the discard pile, subtract 10, and then

subtract off the two ranks that you can see, x and y , you get the rank of the mystery card z !

3.3 Trick 2: Jack the Bounty Hunter

Jack the Bounty Hunter is a trick that you can get extremely creative with by telling a different story every time the trick is performed. To begin select any Jack from the deck of 52 cards; once you have selected your Jack put the card to the side. Now deal two piles of 15 face down across from each other, one towards the participant and the other towards the magician. Have a participant cut both of the decks once each. When the participant cuts the deck, set the cards on the side of the other card. Now resulting in 4 piles, two piles on the participant's side (P1 & P2) and two piles on the magician's side (M1 & M2).

Now have the participant select one of the remaining cards that are not in the four piles. This card will be referred to as the "bad guy." Have the participant look at the card that has been selected and even show it to the audience. Direct the participant to put the "bad guy" on top of one of the piles in front of them face down. With the pile of cards diagonal from the pile the participant chose to put the bad guy on top of, put the cards diagonally across on top of the bad guy. The participant should still have two piles (P1 & P2). The magician should only have one pile. Set the Jack in the magician's pile face up. With the pile diagonal from the magicians pile, place it on top of the Jack. Now the magician and the participant each have one pile. Take the magician's pile and place it on top of the participant's pile, creating one pile.

Now the magician should pick up the whole pile and begin to deal the cards back and forth into two piles. After the pile has been arranged into two piles, you will observe that Jack went into one of the two piles, you will see the Jack because it is the only card face up. The pile that does not have Jack becomes a discard pile. The magician should then pick up the pile that has the Jack, and distribute the pile into two piles, back and forth. Continue to do this until there is only one card in

the pile with Jack. Tell the participant and the audience “Jack has found the bad guy!” Proceed to have the participant turn over the “bad guy” card and watch the audience be amazed.

– Why it works –

Initially there are two piles of 15 cards, call these piles A and B . After the participant cuts the piles, there will be obtain piles A_1 and A_2 which collectively have 15 cards, and piles B_1 and B_2 which collectively have 15 cards.

If the participant places their mystery card on the pile A_i then the magician places the Jack on the pile B_i , and vice versa. The magician then collects the piles so that there are 15 cards between the mystery card and the Jack. For example, if the mystery card was placed on A_1 then the Jack will be on B_1 and the piles will be picked up so that afterwards the piles are stacked in the order B_1, A_2, A_1, B_2 . Then the cards separating the Jack and the mystery card are A_1 and A_2 , which total 15 cards.

Since there is an odd number of cards between the Jack and the mystery card, when the magician picks up the cards and deals back and forth into two piles those two cards will end up in the same pile. Moreover, the number of cards separating them will be 7. After the next dealing into two piles, the number of cards between them will be 3. Then 1. Then 0.

Furthermore, since the total number of cards each time is cut in half, when 7 cards separate them there is a total of 16 cards; when 3 separate them there is a total of 8 cards; when 1 separate them there is a total of 4 cards; when 0 separate them there is a total of 2 cards. So at the end they will be the only remaining cards and will be side by side.

3.4 Trick 3: Guessing Game

This trick does not involve the 10s, Jacks, Queens, and Kings. Therefore make sure they are removed from the pile. Holding this smaller deck face down, have a participant choose any card. We will be using the number on this card, which is called its rank. Tell the participant to multiply the card's rank by 2. Then add 5 to their answer. Now with this new number, multiply it by 5. Have the participant remember this number, because now they will choose their second card from the pile. Have the participant add the number in their head to the rank of their new card. Finally, ask for the participant to tell you and the crowd the number.

The magician then takes this number and subtracts 25 from from it. For example, if they tell you 48. You will then have the number 23. The magician tells the audience you have a 2 (the number in the tens place) and a 3 (the number in the ones place).

– Why it works –

Suppose the value the participant begins with is A ; this is one of the numbers the magician will eventually be able to deduce. After you have them multiply their number by 2 they will get $2A$, and then after adding 5 they will get $2A + 5$. Then, at last, you have the participant multiply by 5, which results in $10A + 25$. With this value the participant then chooses another value B , which the magician will soon be able to guess. You then have them add B to the previous answer, which gives $X = 10A + 25 + B$. This value of X is what the participant says aloud. By taking this number and subtracting off 25 you are left simply with $10A + B$, from which the tens place tells you the first card and the ones place tells you the second card.

3.5 Trick 4: Henry Christ's Improvement

Have an audience member shuffle the deck of cards. Once the magician receives the deck of cards back from the audience, count out nine cards face down into a pile. With these nine cards, have an audience member select a card and look at it, show the rest of the audience, and then place it on top of the pile of eight cards that it came from. Now place the rest of the deck on top of the nine cards.

With the cards in the pile, begin to count from the top of the deck counting pile of 10 backwards, such as 10, 9, 8, ..., 1. While you are counting, if you say the number of the card that you are putting down, then stop counting there; for example if when you are saying the number 8 you are also putting down an 8, stop counting there. For the piles in which the magician is able to count all the way down to 1 without dealing the same card, top that pile with face down card from the deck in hand. Continue this process until there are four piles. Now have an audience member take the remaining deck. With the four piles, focus on the piles which were not capped off. Take the values of these piles and add them together; suppose you get an answer of n . Give the remaining deck to the participant and tell them to find the n^{th} card in the deck. This will be their chosen card.

Note: If all decks are capped, then tell the participant that the cards face down count as a zero. Therefore their card should be in the zeroth card, which is the last card that was placed down. This is unlikely to occur, though.

– Why it works –

First note that by placing their chosen card on top of the stack of 8, which is then placed on the bottom of the deck, the chosen card always ends up in the 44th position in the deck.

Next, the dealing procedure. Treat each card that is capping off a pile as a 0. Like with Counting Kings, notice that in each deck the value of the top card plus the number of cards in that deck always equals 11. Therefore if the values of the top cards are w, x, y and z , then the total number of cards in these stacks is

$$(11 - w) + (11 - x) + (11 - y) + (11 - z) = 44 - w - x - y - z.$$

Recall that in the performance we ignored the capped-off cards, while here the cards are treated as having value 0; this of course makes no difference in the above sum. Observe that the above shows that within the seemingly random dealing procedure the $44 - w - x - y - z$ cards already dealt off. Therefore if the participant is told to take off an additional $w + x + y + z$ cards, the person will end up at the 44th card in the deck, and hence at the chosen card.

3.6 Trick 5: A Baffling Prediction

The magician has a participant thoroughly shuffle a deck of cards. The magician then retrieves the deck and writes a prediction on a piece of paper. The piece of paper with the prediction is handed to one of the participants and the participant is told to keep that paper safe. Next the magician places twelve cards from the deck face down on a table and asks one of the participants to choose any four of the cards, which the magician then turns over. The magician then collects the other eight cards and places them on the bottom of the deck.

With each of the four cards, count from the number on the card to ten. Each time a card is counted, place the card face up from the top of the deck on that pile. For example, if one of the initial four chosen cards was a 6, then the magician will place four cards on top of it, one at a time, while counting aloud 7, 8, 9, 10. If one of the participant's cards is a royal suit, then have an audience member choose an alternative value for that card from 1 to 9, and continue as before.

Lastly, the magician points out the 4 original cards, adds up their values, gives a participant

the deck of cards and asks them to count to that numbered position in the deck. The magician then claims that the chosen card cannot be figured out, but it already has been at the very beginning of the trick. The magician then retrieves the piece of paper, carefully unfolds it, and reveals the prediction, which matches the obtained card.

– **Why it works** –

This trick follows a very similar mathematical process to Henry Christ's Improvement, but with one small, sneaky move. After the deck is shuffled, when the magician takes it back she casually glances at the bottom card; this is the card she writes down as her prediction.

This card is currently in the 52nd position in the deck. As soon as the card is able to be moved to the 44th position the trick will go through as before.

When the twelve cards are placed on the table, four are selected and the other eight are placed on the bottom of the deck. In doing this, the selected card is now in essence in the 44th, and the top four cards are left on the table. From here the trick continues just like in the explanation of Henry Christ's Improvement.

4 Discussion

The researcher's first goal was to review the literature to determine whether there is evidence that doing math-based card tricks can help students learn. Indeed, there is much evidence that they can. It has been shown that there is a need to improve attitudes towards math and math anxiety, and properly using recreational mathematics is a research-based tool to do this.

To use this knowledge with magic tricks, the researcher came up with an example model for how to incorporate the literature review into performing the magic tricks. First, a key piece in using the magic tricks is to understand what may deter the participants from math, such as their attitudes towards math and their math anxiety. Many participants observe math as a dull subject with few interesting applications, so the magic tricks serve as an excellent way to illustrate this other side of math.

For those who have math anxiety, as well as those who do not, the goal is to create a friendly, thought-provoking space to engage students and encourage them to think about math in these new ways. How the tricks work is usually not obvious, so students will usually need guidance to figure out the underlying math, and after seeing the tricks their curiosity will be piqued and they will be motivated to try to figure it out. It is often advised to let the students talk among their peers and with the magician, suggesting their ideas and encouraging thoughts even if they do not seem to be in the right direction. If the students are stuck, the magician can give helpful hints to suggest new lines of thought.

An additional key piece to using math magic is to incorporate multiple participants into the performance, as this can increase engagement with the audience. The magician will get more comfortable with the tricks as they are practiced or performed in front of an audience. Incorporating the audience can be done in very simple ways such as: having an audience member shuffle the cards, having different people choose cards and work together, and creating an interactive story to go along with the tricks (e.g. tell a story to go with the Jack the Bounty Hunter trick).

The last key component when performing a trick is to explain why the trick works. Explaining why the trick works will allow the participants to see and understand the relevant mathematical concepts. The magician can show the trick, let the participants think and discuss their ideas, and then do the trick again to illustrate the mathematical principle in a careful step-by-step demonstra-

tion.

Above we have discussed five tricks: Counting Kings, Jack the Bounty Hunter, Guessing Game, Henry Christ's Improvement, and A Baffling Prediction. Each of the tricks can be used in classes in middle school, high school, or college. Some can be used in multiple classroom settings.

Counting Kings utilizes addition and subtraction to complete the trick, and with the guidance of an educator it could be used as early as elementary school. It would be particularly fitting in a middle or high school algebra class since it involves modeling a problem with equations. These concepts can also be found in introductory courses on college campuses. At California State University, Sacramento it would fit well in the lower division algebra-based courses, including Math 1, Math 9, Math 11 and Math 26A.

To understand the math behind Jack the Bounty Hunter, one must only understand the concept of even versus odd numbers, because the odd number of cards is what makes the Jack and the card the participant selected end near each other. That said, the application of this basic idea is a little subtle, and requires some mathematical maturity to thoroughly grasp. A student should also understand what a variable is because the piles of cards are modeled using variables in the problem, since we do not know the exact number of cards in each pile. Based on the Common Core Standard, adopted by California in June 2010, students begin working with one variable in the sixth grade. Then in the seventh grade students will begin working with multiple variables. This trick can be introduced to students in the sixth grade and can better understand the math in the seventh grade. Going into the college level setting variables are discussed in lower division math such as Math 1, 9, and 11 at Sacramento State. Variables are also discussed on a more complex level in later courses such as Math 32, 105, and 130A.

As in Jack the Bounty Hunter, variables are used in the Guessing Game Trick. Variables are used to model and then solve a system of equations. To begin the trick, there are three variables:

A, B, and X. By the end of the trick there will only be two variables, A and B, the variables being solved for. The variable X is replaced with the number that the participant tells you. This follows the Common Core Standard for the seventh grade. This trick would also be a very simplistic way to use variables for students in higher level math course such as Math 32, 105, and 130A. In a college course where the students have most likely already learned the math in previous years, the trick can be used as a reminder of the material. Furthermore, in higher mathematics a central and recurring learning outcome is to be able to take a problem, determine what is going on, and then, even if the mathematics itself is not particularly difficult, to model and explain it thoroughly. So such a trick could again be useful as an exercise in an upper-level class like Math 108 or Math 101.

5 Future Research

Future research should involve human trials, utilizing a pre- and post-survey. The study should include students from remedial math courses because these students very commonly exhibit math anxiety and distasteful attitudes towards math. It would be interesting for the survey to request demographic information and their interest in math when it is taught in the traditional lecture-based style. The post-survey can serve as a way to gauge whether the magic tricks helped improve the participants' views of math.

The research can also be used to study whether students have a better understanding of the mathematical concept after it has been demonstrated to them in this new way. With the data gained from this, math educators and researchers of math education will have an even better sense of how to use math magic in the classroom to more effectively teach and improve interest in mathematics.

6 Conclusion

Recreational math is another form of math that has been recently implemented in classes. There are multiple forms of recreational math and math magic is just one of them. Math magic has not yet been studied in classes, but based on the previous recreational math that has been used in classes there is evidence that shows math can be taught in multiple ways. In this research a model was formed composed of four steps: 1) understanding what deters the participant 2) creating a thought provoking space 3) incorporating multiple participants and 4) explaining why the trick works.

Within the time frame, the researcher was unable to use the model with students. The model was based on previous research and combines concepts that were found across different pieces of literature that would help students learn math. Since the model was not used in the classroom, there are not findings into how well the model works.

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