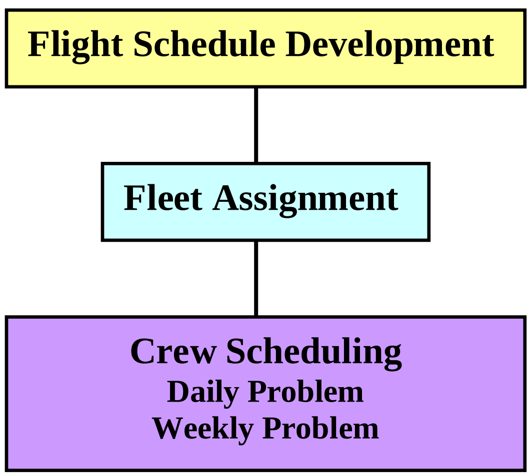
- **Airline Scheduling: An Overview**
- **Crew Scheduling**
- **Time-shared Jet Scheduling (Case Study)**

Airline Scheduling: An Overview



Flight Schedule Development

Flight Number	Departure location time	Arrival location time	Aircraft Type
.	.	.	.
.	.	.	.
.	.	.	.

Given:

1. **Demand and revenues for every origin-destination pair ("market") over time-of-the-day and day-of-the-week**

2. **Route information**
   - **distances**
   - **times**
   - **operating restrictions**

3. **Aircraft characteristics and operating costs**

4. **Operational and managerial constraints**

Find:

A set of flights with
- **departure and arrival times**
- **aircraft assignment**

which maximize profits

Other issues in airline operations:

- **Concurrent flows of *passengers, cargo, aircraft and flight crews* through time**
- **Aircraft maintenance**
- **Management of ground resources:**
    - ***ticketing, check-in, baggage drop-off, gates***

Fleet Assignment

- **Fleet:**
  Group of flights confined to a specific aircraft type

Example:

US Airways typically flies about 2,500 jet flights to over 100 domestic, Caribbean and European markets using more than 400 aircraft of 14 different types

- **Assign an aircraft type to each flight in the schedule**
- **Objective: maximize revenue by**
    - **matching seat capacity to passenger demand**
    - **reducing costs such as fuel, maintenance and airport gating**
- **Requirements:**
    - **restrictions on the operating ranges of aircraft**
    - **curfews and runway limitations at airports**

- **aircraft must stay overnight at stations where maintenance work can be performed**
- **there must also be enough time for passengers to deplane and enplane and for servicing the aircraft**

Today most major airlines use automated procedures based on mathematical optimization models to solve this problem.

At US Airways, the Operations Research Group has been providing automated decision support for the fleeting of schedules since 1993, at an annual benefit to the airline of several million dollars.

Crew Scheduling

- **Pairing:**
  Sequence of flights that start and end at a crew home base

- **Partition a given flight schedule into pairings so that each flight is covered by exactly one crew trip**

WHAT MAKES THE PROBLEM DIFFICULT?

- **Constraints due to crew work rules and FAA safety regulations**

- **Cost of a pairing depends on complex crew pay guarantees**

- **Number of possible pairings is extremely large for many airlines**

Crew Assignment

- **Input: A set of crew trips**
- **Each trip will operate over a range of dates and days of the week**
- **Trips are grouped into monthly flying assignments**
- **The assignments are posted for bid by crew members**

Crew Scheduling: Formulation

Generate pairings such that:

1) **Each pairing starts and ends at a crew home base**
2) **Each pairing conforms to work rules of the airline and FAA safety regulations**

Calculate cost of pairings based on:

1) **Crew salary structure and work rules**
2) **Hotel and other expenses as a result of layovers**
3) **Ground transportation**

pay and credit: number of hours for which a crew member is paid - number of hours he actually flies

Sources of large pay and credit:

1) ***Layovers:*** **Staying at a city other than the home base**
2) ***Deadheading:*** **Transporting crew members as passengers**
3) **Long sit periods between flights**
4) **Short duty days**

- **Problem:** ***Choose a set of pairings with minimum cost such that each flight leg is covered by exactly one pairing***

- **Given m flight legs, n pairings with cost c_j for pairing j**

NOTE THAT n IS USUALLY VERY LARGE AND A FLIGHT LEG MAY BE PART OF MANY PAIRINGS

- **Let a_{ij} = 1 if leg i is part of pairing j and 0, otherwise**

This defines a matrix A = [a_{ij}], where row i corresponds to flight leg i and column j corresponds to pairing j

NOTE THAT A IS PART OF INPUT DATA

-
- **Define decision variables x_j □ {0,1} such that**
  x_j = 1, if pairing j is selected, and 0 otherwise

- **Constraints: for i=1,...,m**

$$\sum_{j}^{n} a_{ij} x_j = 1$$

- **Objective: Min** $\sum_{j}^{n} c_j x_j$

The problem can be formulated as an integer program:

Min $c_1 x_1 + c_2 x_2 + ... + c_n x_n$
st

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = 1$$
$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = 1$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = 1$$
$$x_j \in \{0,1\} \quad \textbf{for} \quad j = 1,\ldots,n$$

Example:

Partial flight schedule:

Flight i	Origin	Destination	Departure time	Flight time
1	NY	BSTN	15	60
2	SF	DNV	40	130
3	PGH	LA	125	195
4	DNV	NY	385	200
5	LA	SF	650	85

Flight times between cities:

City	BSTN	DNV	LA	NY	PGH	SF
BSTN	-	220	250	60	100	245
DNV	220	-	120	200	180	130
LA	250	120	-	240	195	85
NY	60	200	240	-	75	230
PGH	100	180	195	75	-	190
SF	245	130	85	230	190	-

Some of the feasible pairings:

Pairing j	Route	Cost c_j
1	SF -DNV-NY-SF	560

2	SF-DNV-LA-SF	335
3	PGH- LA-PGH	420
4	PGH- LA-SF-PGH	470
5	NY-BSTN-SF-NY	545
6	SF-NY-BSTN-LA-SF	660
7	NY-BSTN-DNV-NY	490

Constraint Matrix A:

Pairing :	1	2	3	4	5	6	7
Flight 1					1	1	1
Flight 2	1	1					
Flight 3			1	1			
Flight 4	1						1
Flight 5		1		1		1	

Min $560x_1 + 335x_2 + 420x_3 + 470x_4 + 545x_5 + 660x_6 + 490x_7$

s.t.

$x_5 + x_6 + x_7 = 1$

$x_1 + x_2 = 1$

$x_3 + x_4 = 1$

$x_1 + x_7 = 1$

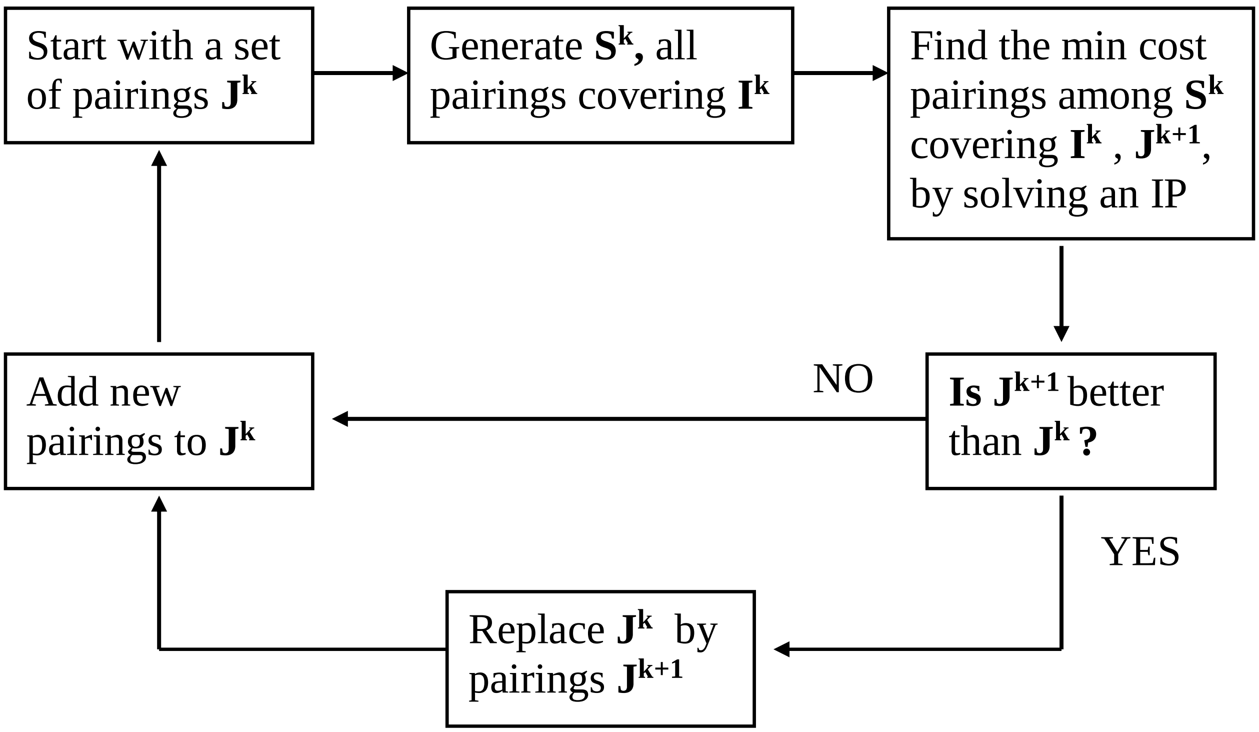
$x_2 + x_4 + x_6 = 1$

$x_1,...,x_7 \in \{0,1\}$

- **This problem is a *set partitioning* problem and is known to be NP-hard**

- **Usually branch and bound with column generation is used**

- **Let J^k be a set of pairings at iteration k and I^k be the set of flight legs covered by J^k**



- **Stop when all legs are covered and a time or iteration limit is reached**

- **Key issues:**
  1. **addition of new pairings (random performs well!)**
  2. **column generation - tree search must be done efficiently**

Flight Scheduling in the Time-Shared Jet Business

Corporate High Flyers

Cannot afford a private jet? How about a quarter of it?

As popularity of elite aviation reaches new heights, fractional ownership programs makes private planes more affordable ...

COMMERCIAL AIR'S GROWING PAINS:

- **Delays, cancelled flights**
- **Being "bumped" from a flight due to overbooking**
- **No direct flights between certain cities**
- **Long connection times**
- **Long check-in times**
- **Misplaced baggage**
- **Lack of enough first class or business class seats**

A PRIVATE PLANE IS AN ALTERNATIVE:

- **Savings in time**
- **Flexibility and convenience**
- **Comfort and privacy**

IF YOU CAN AFFORD IT...

- **High cost ($30 million for a Gulfstream jet)**
- **Operation and maintanence expenses**

HOW ABOUT SHARING A PLANE?

- **All the benefits of private flying**
- **Without the high cost of complete ownership**
- **Without in-house maintenance staff and pilots**

HOW DOES IT WORK?

- **You purchase a portion of an aircraft based on the number of actual flight hours needed annually**
  1/16 share provides 50 hours flying time per year
  1/4 share provides 200 hours of flight time per year

- **You have access to the aircraft any day of the year, 24 hours a day, with as little as four hours notice**

WHAT ARE THE COSTS?

1. **One-time purchase price for the share**
2. **Monthly management fee (maintenance, insurance, administrative and pilot costs)**
3. **Hourly fee for the time the aircraft is used**

Example:

1. **Price of 1/8 share of a Gulfstream IV-SP jet = $4.03 million**
2. **Management fees = $20,500 per month**
3. **Hourly rate = $2,890**

Note:

1. **Ownership rights usually expire after 5 years**
2. **Full ownership is cost-justifiable when annual flight hours exceed 400**

Fractional Jet Ownership programs:

1. <u>NetJets</u>: http://www.netjets.com

"Executive Jet's industry-leading program of fractional aircraft ownership, offers companies and individuals all the benefits of private flying at a fraction of the cost."

"The NetJets fleet of aircraft provides you or your company with efficient access to more locations, increasing the business and personal productivity of key personnel. It's a more affordable alternative for individuals and companies whose flying habits don't justify the cost of an entire aircraft."

2. <u>Flexjet</u> http://www.flexjet.com/new/flex_home2.html

Business is growing

- **In the last 4 years, Executive Jet has ordered 500 new aircraft - 1/3 of the total business jets sold worldwide, totaling over $8 billion**
- **Executive Jet revenues were projected at $900 million for 1998 with an average 35% increase annually**
- **Introduced in May 1995, the Flexjet program has over 350 clients, growing at 100 new fractional owners per year**
- **Raytheon Travel Air program was started in 1997 and currently has more than 300 clients**

WHO ARE THE CUSTOMERS?

- **Small - midsize companies who cannot justify the cost of an aircraft**
- **Corporations who supplement their flight departments**
- **Individual owners range from entrepreneurial CEO Jim McCann to golfers Tiger Woods and Ernie Els**

Case study: Flight Scheduling at Jet-Share Co.

- **Jet-Share Co. owns 4 Lear 30 and 3 Lear 60 aircrafts**

COSTS TO THE CLIENT:

1. Purchase prices

1/8 share of Lear 30 = $1.2 million
1/8 share of Lear 60 = $1.5 million

2. Monthly fees

$5,000 for Lear 30
$6,500 for Lear 60

3. Hourly fees

$1,800 for Lear 30
$2,200 for Lear 60

PROBLEMS WITH OPERATIONS IN THE FIRST 6 MONTHS:

- **Unable to pickup customers on-time 7 times**
- **Subcontracted more than 10 trips**

SCHEDULING AIRCRAFT TO TRIPS (DAILY):

- **At any time, the aircraft are at different locations or are serving a customer**

- **New customer requests arrive**

  origin destination departure time

- **POSITIONING LEG (EMPTY FLIGHT): Relocate an aircraft from its current location to the departure location of the next trip**

- **Every customer request must be satisfied on time, possibly by subcontracting extra aircraft**

- **cost of subcontracting an aircraft for one hour is about ten times the cost of flying an aircraft which is in their fleet**

MAJOR TYPES OF COSTS:

1. **operating costs (fuel, maintenance, etc.)**
2. **penalty costs for subcontracting extra aircraft**

MAIN PROBLEM

Construct a flight schedule with minimum cost
s.t.

1. **all customer requests are satisfied**
2. **maintenance requirements**
3. **previously scheduled trip constraints**

OBJECTIVE: minimize the number of empty flight hours and subcontracted hours

MAINTENANCE CONSTRAINTS:

- **Each aircraft has a specified available flight hours after a periodic maintenance until the next one**
- **Each aircraft can do only a limited number of landings before its next maintenance**

PRE-SCHEDULED TRIP CONSTRAINTS:

- **Trips already assigned to an aircraft should remain so**

SCHEDULING HORIZON: 1-3 days

- **schedule is updated twice daily based on new information**
- **schedules for different types of aircraft are generated separately**

Example

Requested trips 1,...,8 for a given day between locations 1,...,10

Current locations of the aircraft:

Lear 30	Location
1	6
2	7
3	2
4	4

- **Only aircraft 1 has maintenance restrictions**
  1. **it can fly at most 337 minutes**
  2. **it can land at most 9 times before its next maintenance**

- **The information about the trips and travel times between locations are given in the case**

DECIDE: Which trips can be served by each aircraft and ...?

The schedulers create two matrices:

- **AT (Aircraft - Trip) and TT (Trip - Trip)**

AT(i,j) = 1, if aircraft i can serve trip j, and 0 otherwise

TT(j,k) = 1, if trip k can be served *immediately* after trip j by the same aircraft, and 0 otherwise

HOW COULD YOU MODEL THE PROBLEM AS A LINEAR INTEGER PROGRAM?

Define variables:

S_j = 1, if trip j is subcontracted
0, otherwise

Z_{ijk} = 1, if aircraft i serves trip j just before trip k
0, otherwise

for i,j,k such that AT(i,j)=1, AT(i,k)=1 and TT(j,k)=1

In order to represent the number of empty flight hours from the initial location to the departure location of the first trip, let us use a dummy trip, namely trip 0. Then,

Z_{i0k} = 1, if aircraft i serves trip k first
0, otherwise

for i=1,...,n and AT(i,k)=1

Z_{ij0} = 1, if aircraft i serves trip j last
0, otherwise

for i=1,...,n and AT(i,j)=1

Z_{i00} = 1, if aircraft i does not serve any trips
0, otherwise

for i=1,...,n

You should have constraints to ensure that

1. **Each unscheduled trip will either be served by one aircraft, or will be subcontracted**

2. **If an aircraft serves trip k after trip j, trip j is either the first trip served by this aircraft, or it is served after another trip, say trip p**

3. **Each aircraft has a first trip or does not serve any trips at all**

Maintenance restrictions:

4. **Total flight hours of aircraft i is at most the available flight hours before the next maintenance**

5. **Total number of landings of aircraft i is at most the available landings before the next maintenance**

Pre-scheduled trips:

6. **If trip k is pre-scheduled to aircraft i, then it will be served by aircraft i**

- **Maintenance restrictions and pre-scheduled trips make the problem difficult**

- **If these restrictions are relaxed, the problem can be solved efficiently**

A heuristic approach:

1. **Solve the problem without pre-scheduled trips and maintenance restrictions**

2. **Use the given solution to construct a solution for the original problem**