MATH 134: §2.3, #13

MICHAEL VANVALKENBURGH

I used a method that is *very* important in certain branches of math, related to what is called "The Method of Stationary Phase." (Actually, what we have is *non*-stationary phase.) It is basically just integration by parts.

The hardest part is showing that the "middle" integral goes to zero:

$$\int_0^{\pi/4} \exp\left[iR^2 e^{2it}\right]\, iR\, e^{it}\, dt.$$

But we can cleverly rewrite this as

$$\int_0^{\pi/4} \frac{1}{(-2R^2 e^{2it})} \frac{d}{dt}\left[\exp\left(iR^2 e^{2it}\right)\right]\, iR\, e^{it}\, dt = \frac{-i}{2R} \int_0^{\pi/4} \frac{d}{dt}\left[\exp\left(iR^2 e^{2it}\right)\right]\, e^{-it}\, dt.$$

Now use integration by parts to move the derivative to the second factor. Then you can take the limit as $R \to \infty$.

The book has another method, simply using the inequality

$$\frac{x}{\pi} \leq \sin x \qquad \text{for } 0 \leq x \leq \frac{\pi}{2}.$$

Actually, you can get the sharp lower bound

$$\frac{2x}{\pi} \leq \sin x \qquad \text{for } 0 \leq x \leq \frac{\pi}{2}$$

which you can see by drawing the graphs. This inequality is used again in §2.6 (p.158).

[Yes, the book has typos! For another typo, see the discrepancy between the answers to #5 and #7.]