THE N COMMANDMENTS (FOR MATH 31—CALCULUS II)

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This is a list of N topics (currently $N = 29$) from Math 30 that will help you in Math 31. If you study this list, you will have the pleasure of seeing the topics recur throughout the semester. If you forget these topics, you will regularly think, "How did he expect us to remember that?"

On the class webpage, you can find the LaTeX code for this handout, so you can modify it on your own: feel free to delete items that are obvious to you or to add items that you find tricky (but somehow I omitted).

[I am omitting some technicalities, like the fact that $\int_a^b f(x)\,dx$ is not defined for certain functions.]

(1) A circle of radius r has circumference $C = 2\pi r$.

(2) The area enclosed by a circle of radius r is $A = \pi r^2$.

(3) The definitions of sine and cosine in terms of right triangles.

(4) The graphs of sine and cosine.

(5) The interpretation of the derivative as a slope.

(6) $\frac{d}{dx}\cos x = -\sin x$ and $\frac{d}{dx}\sin x = \cos x$.

(7) $\cos^2 x + \sin^2 x = 1$. (Pythagorean Theorem.)

(8) $\cos(2x) = \cos^2 x - \sin^2 x$.

(9) $\sin(2x) = 2\cos x \sin x$.

(10) The idea that you can use the above trig identities to generate even more trig identities.

(11) Usually $(a+b)^2 \neq a^2 + b^2$. (They are equal if and only if $a = 0$ or $b = 0$.)

(12) $\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a+b}$. For example, $\frac{1}{2} + \frac{1}{2} \neq \frac{1}{4}$.

(13) Summation notation (or "Sigma notation"):

$$\sum_{j=1}^{n} a_j$$

is a fancy way of writing $a_1 + a_2 + \cdots + a_n$.

(14) Indefinite integrals: the symbol $\int f(x)\,dx$ represents all antiderivatives of f, that is, all functions F such that $F'(x) = f(x)$.

(15) Definite integrals: for fixed numbers a and b, the symbol $\int_a^b f(x)\,dx$ represents a single *number*.

(16) The Fundamental Theorem of Calculus, Part I:
If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$. (This helps explain the notation used for indefinite integrals.)

(17) The Fundamental Theorem of Calculus, Part II:

$$\int_a^b F'(x)\,dx = F(b) - F(a).$$

(18) $\frac{d}{dx}e^x = e^x$.

(19) The Chain Rule: the derivative of the function $h(x) = f(g(x))$ is

$$h'(x) = f'(g(x))g'(x).$$

(h is "f composed with g", obtained by applying g first, then f.)
For example, if $h(x) = e^{x^2}$, then $h'(x) = 2x\,e^{x^2}$.

(20) The idea that you can use the Chain Rule to implicitly differentiate inverse functions.

(21) The function $g(x) = \ln x$ is the inverse of $f(x) = e^x$. (So one often writes $y = e^x$ and $x = \ln y$.)

(22) $\frac{d}{dx}\ln x = \frac{1}{x}$.

(23) $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.

(24) For any a and $x > 0$ we have

$$\frac{d}{dx}x^a = \frac{d}{dx}e^{a\ln x} = ax^{a-1}.$$

(25) For any a and $x > 0$ we have
$$\frac{d}{da}x^a = \frac{d}{da}e^{a\ln x} = \ln x\, e^{a\ln x} = x^a \ln x.$$

(26) The Product Rule: the derivative of the function $h(x) = f(x)g(x)$ is
$$h'(x) = f'(x)g(x) + f(x)g'(x).$$

(27) The idea that you can use the various "rules" above to figure out the derivatives of all trig (and inverse trig) functions.

(28) How to find all values of x that solve the equation
$$ax^2 + bx + c = 0.$$
(x depends on a, b, and c.)

[A way to figure it out from scratch: "Complete the square."]

(29) The meaning of
$$\lim_{n\to\infty} a_n = L.$$
Here is the definition we will use later in the course:

Let (c, d) be some open interval. We say that a sequence $\{a_n\}_{n=1}^{\infty}$ is *eventually* in (c, d) if there exists some N such that a_n is in (c, d) for all $n \geq N$. (The terms $a_1, a_2, \ldots, a_{N-1}$ may or may not be in (c, d), but all the rest *are* in (c, d).)

We say that the sequence $\{a_n\}_{n=1}^{\infty}$ *converges* to L (equivalently, that L is the limit of the sequence), if:

For any given $\epsilon > 0$, the sequence $\{a_n\}_{n=1}^{\infty}$ is eventually in the interval $(L - \epsilon, L + \epsilon)$.

That is, no matter how small of an interval we put around L, the sequence is eventually in that interval.