MATH 31, LECTURE 13

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§7.1. Integration by Parts, Continued.
I started by discussing Integration by Parts for Definite Integrals. (See the book.)

Example.

$$\int_1^2 \frac{\ln x}{x^2}\,dx = ?$$

We did it in class. Here's a tip that will help you to check your answer: To be careful, I would suggest doing all the integrals before evaluating at the endpoints. Then you can differentiate to check your antiderivative. In this problem, I would write

$$\int_1^2 \frac{\ln x}{x^2}\,dx = \cdots = \left[-\frac{1}{x}\ln x - \frac{1}{x}\right]_1^2.$$

You can check the antiderivative by differentiating. Then you can plug in the endpoints to get the final answer:

$$\frac{1}{2} - \frac{1}{2}\ln 2.$$

Sometimes Integration by Parts is useful when calculating volumes by the Shell Method:
Example. (Repeated from Lecture 9.) Find the volume of the solid generated by rotating the region bounded by the curves $y = e^x$, $x = 0$, $y = 3$ about the x-axis.

Using the Shell Method, we get the integral

$$V = \int_1^3 2\pi y \, \ln y \, dy,$$

which you can evaluate using Integration by Parts. You can check your answer using the Washer Method.

Example. (#31 in the book.)

$$\int_0^{1/2} \cos^{-1} x \, dx = ?$$

In class we took $u = \cos^{-1} x$, $dv = dx$. Remember that you can use Implicit Differentiation to find the derivative of $f(x) = \cos^{-1} x$. (Draw the graph of $f(x) = \cos^{-1} x$ from -1 to 1. The book means for you to take the y-values between 0 and π. The function is decreasing, so the derivative is a *negative* square root.)

For this particular problem, it's actually easiest to integrate by interpreting the integral as an area, then integrating with respect to y. Then we don't need to use Integration by Parts. It's simply:

$$A = \frac{1}{2} \cdot \frac{\pi}{3} + \int_{\pi/3}^{\pi/2} \cos y \, dy.$$