MATH 31, LECTURE 14

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§7.2. Using Trig Identities to Calculate Certain Integrals.

We started with some warm-up problems: "Which of the following integrals appear amenable to the Substitution Rule?" I gave you time to think about them, then we went through them one-by-one.

The most basic trig identities are

$$\cos^2\theta + \sin^2\theta = 1 \tag{1}$$

and the "double-angle formulas"

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta \tag{2}$$

$$\sin(2\theta) = 2\cos\theta\sin\theta. \tag{3}$$

I don't say this often, but you should memorize these formulas. Remember that the first identity follows from the Pythagorean Theorem. (And the last two can be generalized to the "angle addition formulas.") From these we can derive the "half-angle formulas," by either adding (2) to (1) or by subtracting (2) from (1).

Strategy for evaluating

$$\int \sin^m x\, \cos^n x\, dx \qquad [\ldots \text{see the book.}]$$

Example.

$$\int \sin^4 x\, \cos^5 x\, dx.$$

[We did it in class.]

The method is similar for integrals involving sec and tan. The main identity involving sec and tan comes from (1). Simply divide both sides of (1) by $\cos^2\theta$ to get

$$1 + \tan^2\theta = \sec^2\theta.$$

Also, by the quotient rule,

$$\begin{aligned}\frac{d}{dx}\tan x &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)\\ &= \sec^2 x\end{aligned}$$

and

$$\begin{aligned}\frac{d}{dx}\sec x &= \frac{d}{dx}\left(\frac{1}{\cos x}\right)\\ &= \tan x \sec x.\end{aligned}$$

Note: All of these facts about sec and tan come from facts about sin and cos.

Example. (# 31 in the book.)

$$\int \tan^5 x\, dx.$$

The way the book presents the material, they want you to use a substitution involving tan or sec. But note that all of our facts about sec and tan come from facts about sin and cos. So we could just as well convert the integral to an integral with sin and cos, and that would work just as well:

$$\begin{aligned}\int \tan^5 x\, dx &= \int \frac{\sin^5 x}{\cos^5 x}\, dx\\ &= \int \frac{(\sin^2 x)^2 \sin x}{\cos^5 x}\, dx\\ &= \int \frac{(1-\cos^2 x)^2 \sin x}{\cos^5 x}\, dx\\ &\vdots\end{aligned}$$

Now make a substitution involving sin and cos and you will get the correct answer.

The take-away message: it's just "the Substitution Rule + trig identities."