MATH 31, LECTURE 26

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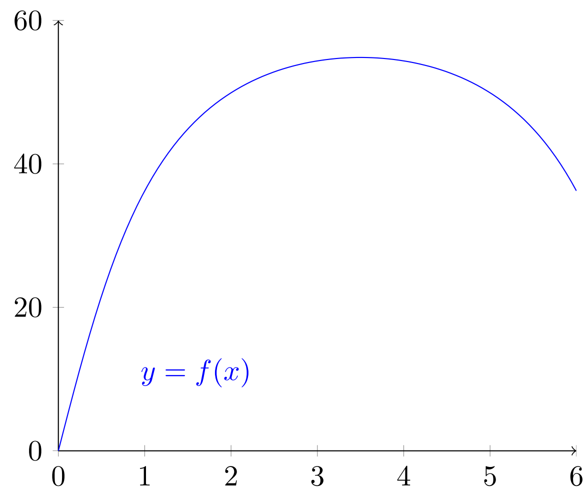
I passed out exams and gave you the following handout to work on:

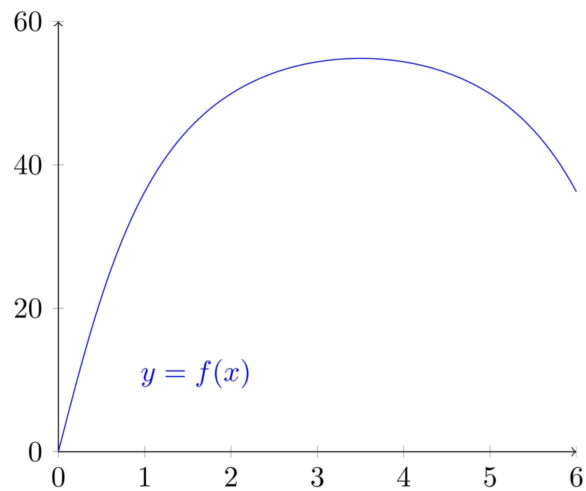
[In hindsight, perhaps I should have given you an example where $\Delta x \neq 1$.]

Sketch an approximation for $\int_0^6 f(x)\,dx$, with $f(x) = \frac{1}{100}\arctan(7-x)\arctan(x)$, by splitting the interval $[0,6]$ into $n=6$ equal subintervals, using each of the four methods from Friday. Write a formula (using the letter f) for each of the methods.

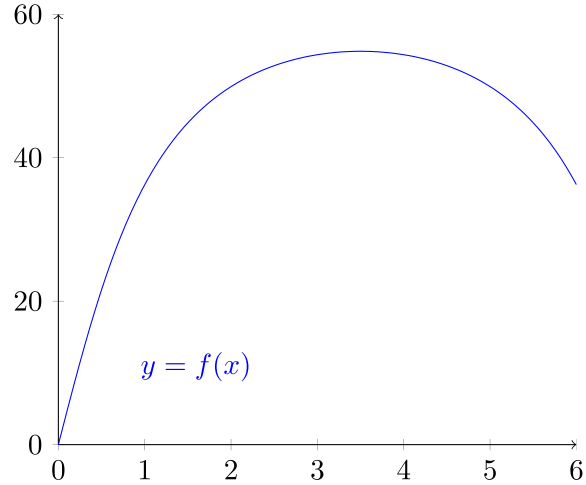
Left endpoint approximation:



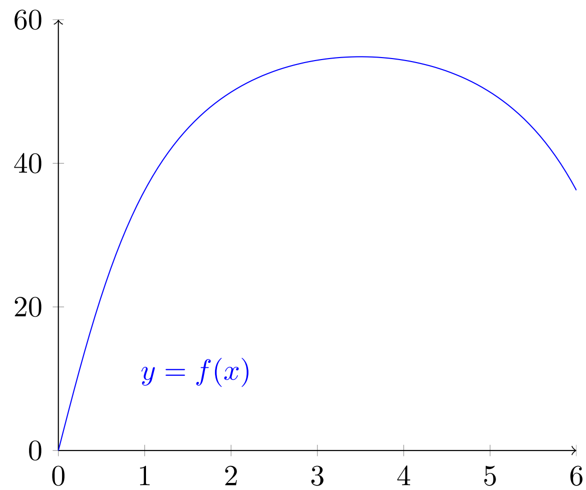
Right endpoint approximation:



Midpoint Rule:



Trapezoidal Rule:



In each of the methods, in the limit as $n \to \infty$ we get the *exact* value of the integral. (See §5.2.) But what if we stop at a finite n (e.g. $n = 6$ in the handout)? Which method is best?

First of all, how do we measure how "good" an approximation is?

Using the left endpoint approximation, last time I wrote

$$\int_a^b f(x)\,dx \approx I_{\text{left}}.$$

It is more precise to write instead

$$\int_a^b f(x)\,dx = I_{\text{left}} + E_{\text{left}},$$

where E_{left} is the *error*. Similarly,

$$\begin{aligned}
\int_a^b f(x)\,dx &= I_{\text{right}} + E_{\text{right}}, \\
\int_a^b f(x)\,dx &= I_{\text{midpt}} + E_{\text{midpt}}, \quad \text{and} \\
\int_a^b f(x)\,dx &= I_{\text{trap}} + E_{\text{trap}}.
\end{aligned}$$

For a given integral, which method has the smallest error?

Example (from last time). When approximating $\int_0^2 x^2\,dx$ with $n = 4$, we found

$$\begin{aligned}
I_{\text{left}} &= \frac{7}{4} \\
I_{\text{right}} &= \frac{15}{4} \\
I_{\text{midpt}} &= \frac{21}{8}, \quad \text{and} \\
I_{\text{exact}} &= \frac{8}{3}.
\end{aligned}$$

And today we figured out:

$$I_{\text{trap}} = \frac{11}{4}.$$

Now which method is the best? That is, which method has the smallest error?

$$
\begin{aligned}
E_{\text{exact}} &= \frac{8}{3} - \frac{8}{3} = 0. \quad :) \\
E_{\text{left}} &= \frac{8}{3} - \frac{7}{4} = \frac{11}{12} \\
E_{\text{right}} &= \frac{8}{3} - \frac{15}{4} = -\frac{13}{12} \\
E_{\text{midpt}} &= \frac{8}{3} - \frac{21}{8} = \frac{1}{24} \\
E_{\text{trap}} &= \frac{8}{3} - \frac{11}{4} = -\frac{1}{12}.
\end{aligned}
$$

So for this example the Midpoint Rule gives the best approximation.