MATH 31, LECTURE 4

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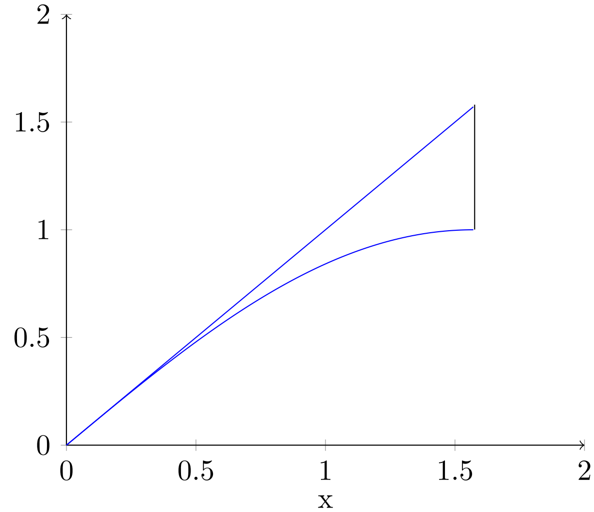
We started class with a ten-minute quiz, with questions from your homework.

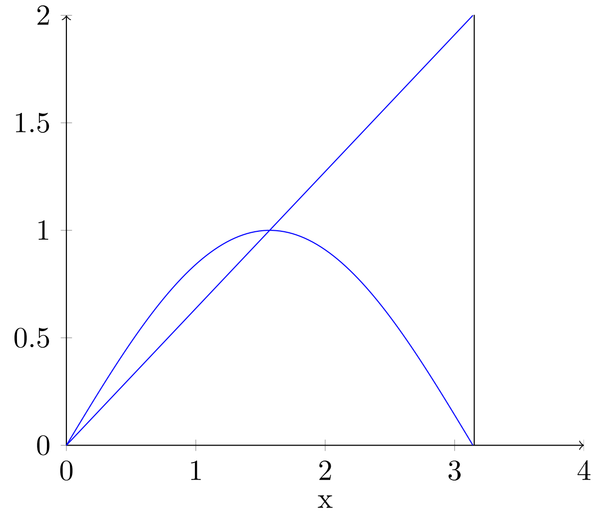
Example. Calculate the area between the curves $y = x$ and $y = \sin x$, between $x = 0$ and $x = \frac{\pi}{2}$.

Answer. We have the picture:



$$\begin{aligned} A &= \int_0^{\pi/2} |x - \sin x| \, dx \\ &= \int_0^{\pi/2} (x - \sin x) \, dx \\ &= [\tfrac{1}{2}x^2 + \cos x]_0^{\pi/2} \\ &= \frac{\pi^2}{8} - 1. \end{aligned}$$

Example. Calculate the area between the curves $y = \frac{2}{\pi}x$ and $y = \sin x$, between $x = 0$ and $x = \pi$.
Answer. We have the picture:

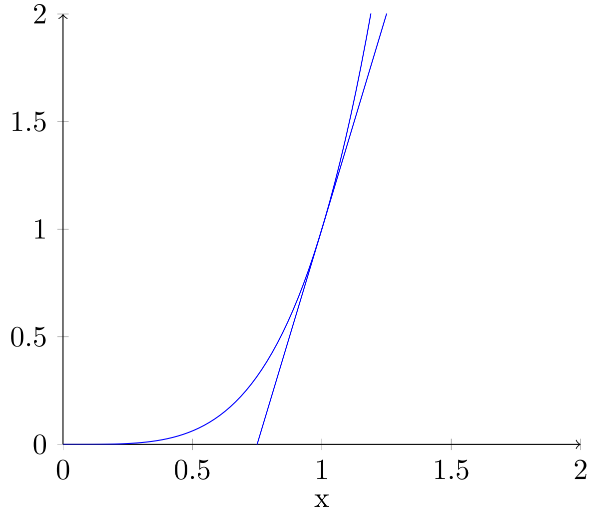


To remove the absolute value in the integral, we need to break it into two parts:

$$\begin{aligned} A &= \int_0^\pi |\sin x - \tfrac{2}{\pi}x|\,dx \\ &= \int_0^{\pi/2} (\sin x - \tfrac{2}{\pi}x)\,dx + \int_{\pi/2}^{\pi} (\tfrac{2}{\pi}x - \sin x)\,dx \\ &\vdots \\ &= \frac{\pi}{2}. \end{aligned}$$

Example. Find the area of the region bounded by the curve $y = x^4$, the tangent line to the curve at $(1, 1)$, and the x-axis.
Answer. [I gave you time to work on it, then I gave you time to talk to your neighbors.] From Calc. I, we know how to find the equation for the tangent line at $(1, 1)$. Let $f(x) = x^4$. Then $f'(x) = 4x^3$, and $f'(1) = 4$ is the *slope* of the tangent line at $(1, 1)$. Thus the equation of the tangent line through $(1, 1)$ is

$$y = 4x - 3.$$



One way to do it:

$$A = \int_0^{3/4} x^4\,dx + \int_{3/4}^1 (x^4 - (4x-3))\,dx$$
$$\vdots$$
$$= \frac{3}{40}.$$

Another way to see it:

$$A = \int_0^1 x^4\,dx - \int_{3/4}^1 (4x-3)\,dx$$
$$\vdots$$
$$= \frac{3}{40}.$$

A different way to do it: In class, some of you had *this* smart idea: Integrate with respect to y! The curves, as graphs of functions of y, are $x = y^{1/4}$ and $x = \frac{1}{4}y + \frac{3}{4}$. Then the integral is:

$$\int_0^1 \left((\tfrac{1}{4}y + \tfrac{3}{4}) - y^{1/4} \right) dy = \cdots = \frac{3}{40}.$$

If you have extra time on an exam, it would be a good idea to solve a problem different ways, and make sure you get the same answer every time.