

MATH 31, LECTURE 5

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I'll be brief—please read the textbook for details.

Before we move on to Section 6.2, let's talk about integrals of even and odd functions. The book discussed integrals of even and odd functions at the end of Section 5.5, but I think it could've been its own short section. (The Substitution Rule and Even/Odd functions don't need to be taught together.)

An evil test question: Evaluate the definite integral

$$\int_{-5}^5 x^7 e^{-x^2} \cos x \, dx.$$

§6.2. Volumes.

The volume of a right cylindrical prism with face area A and height h is *defined* to be $V = Ah$. These are the basic objects in this section.

We approximate more general solids by “sums” of right cylindrical prisms of height Δx . (Analogously, in Section 5.1 we approximated areas under curves by “sums” of simpler objects: rectangles.) See the book for nice pictures and further details.

We calculated the volume of the ball of radius r .

As a next example, we find the volume of the solid generated by revolving the curve $f(x) = \sin x$, $0 \leq x \leq \pi$, about the x -axis. Cutting the solid with the plane at x , the cross-section is a disc of radius $\sin x$. So the cross-sectional area is $A(x) = \pi \sin^2 x$. Hence the volume of the solid is:

$$V = \int_0^\pi \pi \sin^2 x \, dx = \cdots = \frac{\pi^2}{2}.$$