MATH 31, LECTURE 8

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I'll be brief–please read the textbook for details.

Section 6.3.
I started by explaining the theory behind the "shell method."

Example. Find the volume of the solid generated by rotating the region bounded by

$$y = \sqrt{x}, \qquad y = 0, \qquad x = 1$$

about the line $x = 2$.

For $0 \leq x \leq 1$, the typical shell has radius $2 - x$, circumference $2\pi(2 - x)$, and height $\sqrt{x}$. Thus the volume is

$$V = \int_0^1 2\pi(2 - x)\sqrt{x}\,dx = \cdots = \frac{28\pi}{15}.$$

It's good practice (and a good way to check our answer) to also solve the problem using the washer method. The cross-section at y, for $0 \leq y \leq 1$, is an annulus of inner radius 1 and outer radius $2 - y^2$. So the cross-sectional area is

$$A(y) = \pi(2 - y^2)^2 - \pi.$$

Thus the volume is

$$V = \int_0^1 \left[\pi(2 - y^2)^2 - \pi\right]\,dy = \cdots = \frac{28\pi}{15}.$$

I think it's fun to use two different methods to calculate the volume. We get two different-looking integrals that are actually *equal*!