# MATH 31, LECTURE 9

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**Example.** [In class I drew a nice picture.] Find the volume of the solid consisting of discs stuck between the $x$-axis and the curve $y = x^2$. [It makes sense if I draw the figure.] I wanted to give you an example of a solid that is *not* a solid of revolution. You can, however, do a clever trick (look up "Cavalieri's Principle") to get a solid of revolution with the same volume.

For $0 \le x \le 2$ the cross-section is a disc of radius $\frac{1}{2}x^2$, so its area is $A(x) = \frac{1}{4}\pi x^4$, and

$$V = \int_0^2 \tfrac{1}{4}\pi x^4\, dx = \cdots = \frac{8}{5}\pi.$$

**Example.** Find the volume of the solid generated by rotating the graph of $f(x) = |x-2|+1$, $0 \le x \le 2$, about the $x$-axis. [I drew the picture in class.]

For $0 \le x \le 2$, the cross-section is a disc of radius $3-x$, so its area is $A_1(x) = \pi(3-x)^2$. For $2 \le x \le 3$, the cross-section is a disc of radius $x-1$, so its area is $A_2(x) = \pi(x-1)^2$. So the volume of the solid is

$$\begin{aligned} V &= \int_0^2 A_1(x)\, dx + \int_2^3 A_2(x)\, dx \\ &= \int_0^2 \pi(3-x)^2\, dx + \int_2^3 \pi(x-1)^2\, dx \\ &= \int_1^3 \pi u^2\, du + \int_1^2 \pi u^2\, du \\ &= 11\pi. \end{aligned}$$

**Example.** Consider the solid generated by rotating the region bounded by the curves

$$y = e^x, \qquad x = 0, \qquad y = 3$$

about the $x$-axis. Set up but do not evaluate the integral for the volume, using (a) the washer method and (b) the shell method.

[We did it in class.] Exercise: Try to do a substitution showing that the integrals actually do evaluate to the same number.