

MATH 45, LECTURE 1

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1. ADMINISTRATION OF THE COURSE

See the class webpage: <http://webpages.csus.edu/mjv/Math45spring16.html>

2. BOOKS

There is no standard text for this class. Some use *Elementary Differential Equations* by Boyce and DiPrima, and others use *A First Course in Differential Equations* by Zill. I decided to use my colleague's (free!) typed lecture notes: *Differential Equations* by András Domokos. I use free books when I can because I would prefer not to force students to buy the latest edition of an expensive book. If you would prefer a traditional book, please go ahead and buy an old edition of Boyce and DiPrima or Zill, and read that, too!

Another book is *Ordinary Differential Equations* by Tenenbaum and Pollard. I bought it on a whim the summer after my freshman year of college and loved it! It is less than \$15, and it is full of cool examples, about money, fighter jets, bacteria, etc.

3. RADIOCARBON DATING

In fact, my first example, for motivation, comes from Tenenbaum and Pollard (page 1):

In 1940, a group of boys hiking near Lascaux, France, discovered a cave with ancient cave paintings and the charcoal remains of a fire.

Question: How long ago did the cave dwellers live?

Fact: All living organisms contain C^{14} , an unstable isotope of carbon. When the organism dies, it stops producing C^{14} , and the C^{14} begins to decompose due to radiation. (That is, as time goes on, there is less and less of it.)

Fact: A chemist tells us that 99.876% of C^{14} present at death will remain after 10 years.

Fact: Let $y(t)$ represent the amount of C^{14} present in the dead tree (that is, the charcoal) at time t . Then the rate of decay of the C^{14} is:

$$(*) \quad \frac{dy}{dt} = -k y,$$

for some constant $k > 0$. This is a quantitative way of saying “the more there is, the faster it decays.” It is a *differential equation*: an equation involving a function $y(t)$ and its derivative(s).

Note: Using the Chain Rule from calculus, we see that equation (*) can be rewritten as:

$$\frac{d}{dt} \ln(y(t)) = -k.$$

We integrate and find that $\ln(y(t)) = -kt + C$, where C is the constant of integration. Exponentiating both sides, we find that $y(t) = e^{-kt+C}$. It is convenient to define $A = e^C$. Then we have

$$y(t) = Ae^{-kt}.$$

This is the general solution of the differential equation (*). The quantity $y(0) = A$ represents the amount of C^{14} at time $t = 0$.

At time $t = 10$ years, according to the chemist mentioned above, we have

$$y(10) = 0.99876 A = Ae^{-10k}.$$

Dividing by A and taking the logarithm, we find that

$$-10k = \ln(0.99876).$$

That is,

$$k \approx 0.000124.$$

So the amount of C^{14} remaining after t years is

$$y(t) = Ae^{-0.000124t}.$$

By chemical analysis of the charcoal, a chemist finds that

$$y(t) = 0.145 A.$$

Thus at time t we have

$$0.145 A = Ae^{-0.000124t}.$$

Again dividing by A and taking the logarithm, we get

$$\ln(0.145) = -0.000124t.$$

That is, $t \approx 15573$ years. So science tells us that the cave dwellers lived about 15573 years ago! (Wikipedia says that the paintings are approximately 17300 years old. Maybe someone tested the paintings rather than the charcoal?)

Willard Libby developed this method in the 1940s and won the Nobel Prize in Chemistry in 1960.

[Then I did a quick review of differentiation: the definition of the derivative, the geometric meaning, examples of derivatives, and rules of differentiation.]