MATH 45, LECTURE 3

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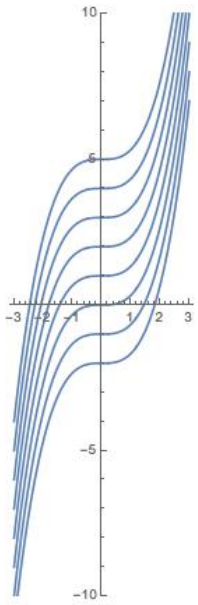
There are infinitely many solutions of the differential equation

$$y'(t) = t^2,$$

but they only differ by adding constants. The general solution is the indefinite integral:

$$\int t^2\,dt = \tfrac{1}{3}t^3 + C,$$

where C can be any constant. Pictured below are a few of the solutions.



But there is only *one* solution of the "initial value problem"

$$y'(t) = t^2, \qquad y(0) = 4.$$

It is the definite integral

$$y(t) = 4 + \int_0^t s^2\,ds = \tfrac{1}{3}t^3 + 4.$$

Can you see it in the picture?