PRACTICE SET FOR MIDTERM 1

Problem 1
Find the natural domain and the horizontal/vertical asymptotes of the following functions

(1) $f(x) = \dfrac{x+1}{x^2}$

(2) $f(x) = e^{\frac{1}{x^2-9}}$

(3) $f(x) = \arctg\left(\dfrac{x^3+4}{x^3}\right)$

(4) $f(x) = \ln\left(\dfrac{3x-1}{x-2}\right)$

(5) $f(x) = \dfrac{\cos(x^2)}{x^2+1}$

Problem 2
Compute the following limits if they exist or prove that they don't exist

(6) $\displaystyle\lim_{x\to+\infty} \dfrac{3e^{4x}+1}{e^x - e^{4x}}$

(7) $\displaystyle\lim_{x\to 0} \dfrac{2 - \sqrt{4+x^2}}{9x^2}$

(8) $\displaystyle\lim_{x\to 0} \dfrac{\frac{10}{3+2x} - \frac{10}{3}}{x^2 - 2x}$

(9) $\displaystyle\lim_{x\to-\infty} \cos(4x)\cdot e^{-x^2}$

(10) $\displaystyle\lim_{x\to+\infty} \sin(x)\cdot \dfrac{x}{x^2 - 2}$

(11) $\displaystyle\lim_{x\to 0} \dfrac{|x^3|}{x^3 - x}$

(12) $\displaystyle\lim_{x\to-1} \dfrac{2x+2}{|x+1|}$

(13) $\displaystyle\lim_{x\to-\infty} \dfrac{\sqrt{x^2+1}}{7x}$ (tricky)

Problem 3
Are the following functions continuous? If not classify the type of discontinuity they exhibit.

(14) $f(x) = \sqrt{x^6 + x^4 + 2}$

(15) $f(x) = |x^2 - 4|$

(16) $f(x) = \begin{cases} \frac{x+1}{x-2} & \text{for } x > 2 \\ x^3 - 1 & \text{for } x \le 2 \end{cases}$

(17) $f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$

Do the following equations admit any real solutions?

(18) $\ln(x - 1) + \ln(x) = 1$

(19) $x^5 - x = 2$

(20) $\text{arctg}(x) = x^3 - x$

(21) $e^x + x^2 + 2 = 0$

Problem 4
Compute the derivatives of the following functions

(22) $f(x) = (x^3 - 3x^2 + 1)\cos(3x^2)$

(23) $f(x) = \dfrac{\text{arctg}(x^2)}{e^{-x}}$

(24) $f(x) = \sqrt[4]{x^6 - 2}$

(25) $f(x) = \arcsin\left(\dfrac{3}{x^3}\right)$

(26) $f(x) = \sin^5(3x)$

(27) $f(x) = (x^5 + 1)^{2x-1}$

Problem 5
Find $\frac{dy}{dx}$ for the functions implicitly defined by the following equations

(28) $x^2 = y^3 - 2x$

(29) $\cos(y) = xy^2 + 2$

(30) $\ln(xy) = x + y$

(31) $\arcsin(y^2) = e^{x+y}$

Problem 6
32) Determine the equation of the tangent line to the graph of the function $y = \ln(x)$ at $x = 1$.
Is there a point where the tangent is parallel to the line $y = 3x - 1$?
Is there a point where the tangent is horizontal?

33) Determine the equation of the tangent line to the ellipse of horizontal semi-axis 3 and vertical semi-axis 4 at the point $(\sqrt{5}, \frac{8}{3})$.
Are there any points where the tangent to the ellipse is horizontal?

34) Is the line $y = -x + 2$ tangent to the graph of the function $y = \ln(x^2 + 1)$?

35*) Suppose $f(x) \le x\cos\left(\frac{1}{\sqrt{x}}\right)$ for all $x > 0$. Can we compute $\lim\limits_{x \to 0^+} f(x)$?
What if $|f(x)| \le x\cos\left(\frac{1}{\sqrt{x}}\right)$ for all $x > 0$?

36**) Does there exist a common tangent to the curves $y = e^x$ and $y = -x^2$?