Math of Elections #6

Pairwise Comparison Method

Definition

The **pairwise comparison method** for elections works as follows. Look at every pair candidates: determine who would win in a one-on-one election; the winner gets 1 point and loser gets 0. If there is a tie, both get 1/2 point. Add up the points from all pairwise comparisons. Candidates are ranked according to the number of total points they received.

Exercise

There is an election with 8 voters and 4 candidates: Alejandro (A), Baobao (B), Corrina (C), and Destiney (D).

Ranking	**Ballot**	**Ballot**	**Ballot**	**Ballot**	**Ballot**	**Ballot**	**Ballot**	**Ballot**
1st	A	B	B	B	B	D	D	D
2nd	C	C	C	C	C	A	A	C
3rd	B	A	A	A	A	C	C	A
4th	D	D	D	D	D	B	B	B

1. Determine who would win each pairwise competition: A vs B, A vs C, A vs D, B vs C, B vs D, C vs D.

2. Use the pairwise comparison method to rank all of the candidates.

3. What would have been the ranking using the plurality method?

4. Compare the outcomes using pairwise comparison verse using plurality. Which do think does a better job of choosing the winner of this election? Why?

Exercise

Back to the Club Election Example. We saw Candy wins using the plurality method, Emma wins using a Borda count, and Nguyen wins using plurality with elimination. Rank the candidates using the pairwise comparison method. Who won? Which of the four methods do think does a better job of choosing the winner of this election?

Number of Voters	**14**	**10**	**8**	**4**	**1**
1st	C	L	N	E	L
2nd	E	E	L	N	N
3rd	L	N	E	L	E
4th	N	C	C	C	C

Exercise

Suppose there is an election using the pairwise comparison method.

1. If there were 5 candidates, how many pairwise comparisons would be needed?

2. If there were 6 candidates, how many pairwise comparisons would be needed?

3. If there were 20 candidates, how many pairwise comparisons would be needed?

4. Can you come up with a formula for how many pairwise comparisons would be needed if there were N candidates? Try it out for $N = 100$.