Math of Elections #8

Fairness & Arrow's Impossibility Theorem

Fairness Criteria

What properties might we want an election method to have so that we feel like the method is "fair"? Some that we have encountered already are listed below. Are there any you would add?

- The **majority criterion**: if there is a majority candidate, they should be the winner.
  - In other words: if a candidate gets the majority of the first-place votes, then they should win.
- The **Condorcet criterion**: if there is a Condorcet candidate, they should be the winner.
  - In other words: if a candidate can beat everyone individually, then they should beat everyone together.
- The **monotonicity criterion**: if a candidate would win, they should still win if a voter moved them higher on their preference ballot.
  - In other words: it should not be possible to hurt a candidate by ranking them higher on the ballot.
- The **Independence of Irrelevant Alternatives (IIA) criterion**: if a candidate would win, then they should still win if any other candidate was removed.
  - In other words: a candidate who is not able to win should not be able to spoil the election for someone who could.

Exercise

Look over our work with the various voting methods. If a method always satisfies a criterion, put a ✓ in the appropriate box below; if a method could possibly violate the criterion, put an ✕ in the box. Justify with a brief sentence or reference to an exercise we did before. Are there any that you're unsure about?

Criterion	Plurality	Borda Count	Plurality with Elimination	Pairwise Comparison
majority				
Condorcet				
monotonicity				
IIA				

Exercise

Reflect on the voting methods we have discussed.

1. What is your favorite method? What are the benefits and drawbacks of that method? *Please be specific and reference the fairness criteria*.

2. Try to imagine a better election method, and briefly describe it. (Perhaps it's an altered version of one we discussed or a hybrid of more than one.) What issues are your method trying to fix?

Arrow's Impossibility Theorem

Arrow's Impossibility Theorem states that if there is an election with at least three candidates, then there does *not* exist any voting method that satisfies all four of the fairness criteria listed above.