

Patterns #3

Sequences of numbers

Definition

A **sequence** (of numbers) is an infinite list of numbers. A specific example is 1, 1, 2, 5, 27, ... and the general form can be written $a_1, a_2, a_3, a_4, a_5, \dots$. Each individual number in the sequence is called a **term**.

Exercise

Consider the following sequence: 3, 10, 17, 24, 31, ...

1. What is the relationship between the 1st and 2nd terms? What about the 2nd and 3rd terms? 3rd and 4th terms? What is the pattern? Use the pattern to determine the 6th and 7th terms of the sequence.
2. Let's find a formula for the terms. Complete the first 5 rows of the table below by filling in the blanks. Then look for a pattern, and use it to determine the *Work* for the 20th term and the value of the 20th term. Finally, repeat for an arbitrary n th term. The *Work* for the n th term is the formula we want.

Term number	Work to compute the term	Value of the term
1	$3 + 7 \cdot \underline{\quad}$	3
2	$3 + 7 \cdot \underline{\quad}$	10
3	$3 + 7 \cdot \underline{\quad}$	17
4	$3 + 7 \cdot \underline{\quad}$	24
5	$3 + 7 \cdot \underline{\quad}$	31
:	:	:
20		
:	:	:
n		N/A

3. Use the formula you found to determine if the number 576 will appear in the sequence.

Exercise

Consider the following sequence: 3, 6, 12, 24, 48, ...

1. What is the relationship between the 1st and 2nd terms? What about the 2nd and 3rd terms? 3rd and 4th terms? What is the pattern? Use the pattern to determine the 6th and 7th terms of the sequence.
2. Let's find a formula for the terms. Like before, fill in the first 5 rows of the table, look for a pattern, and use the pattern to complete the rest of the table. (The *Work* for this sequence will be much different than the previous one.)

Term number	Work to compute the term	Value of the term
1		3
2		6
3		12
4		24
5		48
⋮	⋮	⋮
20		
⋮	⋮	⋮
n		N/A

Exercise

1. Suppose there is a sequence a_1, a_2, a_3, \dots and you know an *explicit* formula for the n th term of the sequence is $a_n = 4 + 5n$. Use this to write the first five terms of the sequence. What's the 100th term?
2. Suppose there is a sequence a_1, a_2, a_3, \dots . Assume you know that $a_1 = 3$ and that a *recursive* formula for the n th term of the sequence is $a_n = a_{n-1} + 2n$. What does this formula say when $n = 2$? Use it to find the second term of the sequence. Continue on to find the 3rd, 4th, and 5th terms. What would you need to do to determine the 100th term when using a formula like this?