Patterns #4

Arithmetic sequences

Definition

A sequence $a_1, a_2, a_3, a_4, a_5, \ldots$ is called **arithmetic** if there is a number d such that each term in the sequence can found by *adding* d to the previous term: $a_1 \overset{+d}{\rightarrow} a_2 \overset{+d}{\rightarrow} a_3 \overset{+d}{\rightarrow} a_4 \overset{+d}{\rightarrow} a_5 \overset{+d}{\rightarrow} \cdots$. This means that $a_n = a_{n-1} + d$ for each index $n \geq 2$. The number d is called the **common difference**.

Exercise

Determine if each sequence is arithmetic or not. If it is arithmetic, determine the common difference.

1. $3, 5, 7, 11, 13, 17 \ldots$

2. $2, 8, 14, 20, 26, 32, \ldots$

3. $10, 3, -4, -11, -18, -25, \ldots$

4. $2, 4, 8, 16, 32, 64, \ldots$

5. $7, 9.25, 11.5, 13.75, 16, 18.25, \ldots$

6. The *number of blocks* in the sequence of figures beginning:

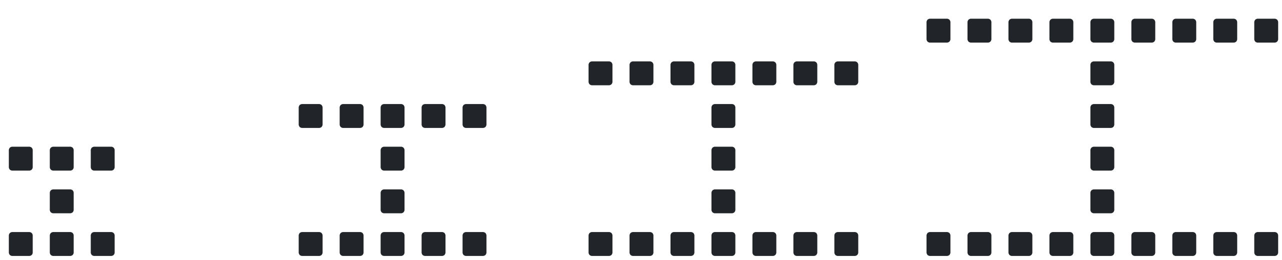

Figure 1 Figure 2 Figure 3 Figure 4

Exercise

Suppose that $a_1, a_2, a_3, a_4, a_5, \ldots$ is an arithmetic sequence with common difference 4. Suppose that you also know $a_2 = 23$. Determine each of a_1 and a_3, and then determine a_{100}.

Arithmetic sequence formula

If $a_1, a_2, a_3, a_4, a_5, \ldots$ is an arithmetic sequence with common difference d, then a formula for the nth term is:
$a_n = a_1 +$__________

Exercise

Suppose that $a_1, a_2, a_3, a_4, a_5, \ldots$ is an arithmetic sequence. If $a_1 = 7$ and $a_{33} = 359$, what is the common difference? *Try using your formula above.*

Exercise

An estimated 100 billion single-use plastic bags are used (and then thrown away) in the US each year. Each shopper uses on average 450 of these bags each year, and if 1% of shoppers stopped using these bags, then about 0.945 billion bags could be subtracted from the 100 billion that end up in the trash each year. Let's study what would happen if an additional 1% of shoppers stopped using these bags each year. Let B_1 be the number of bags in billions that are being used at the beginning of the 1st year of our study, so $B_1 = 100$. Then B_2 is the number of bags being used after the first 1% reduction, so $B_2 = 100 - 0.945 = 99.055$.

1. Determine B_3, the number of bags being used at the start of the 3rd year (after two 1% reductions).

2. Write out an explicit formula for B_n, and use it to compute B_{20}.

3. According to this model, at the beginning of what year would there be no more single-use plastic bags being thrown away?