Patterns #5

Geometric sequences

Definition

A sequence $a_1, a_2, a_3, a_4, a_5, \ldots$ is called **geometric** if there is a number r such that each term in the sequence can found by *multiplying* the previous term by r: $a_1 \overset{\times r}{\to} a_2 \overset{\times r}{\to} a_3 \overset{\times r}{\to} a_4 \overset{\times r}{\to} a_5 \overset{\times r}{\to} \cdots$. This means that $a_n = r \cdot a_{n-1}$ for each index $n \geq 2$. The number r is called the **common ratio**.

Exercise

Determine if each sequence is arithmetic, geometric, or neither. If arithmetic, determine the common difference, and if geometric, determine the common ratio.

1. $3, -12, 48, -192, 768, \ldots$

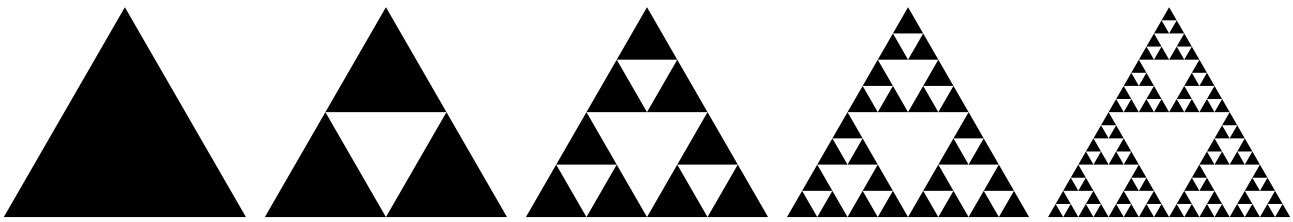
2. $1, 4, 8, 32, 36, \ldots$

3. $13, 8, 3, -2, -7, -12, \ldots$

4. $144, 72, 36, 18, 9, \ldots$

5. $1, 4, 9, 16, 25, \ldots$

6. The *number of black triangles* in the sequence of figures beginning:



By Original: Saperaud Vector: Wereon - Own work based on: Sierpinsky triangle (evolution).png

Exercise

Suppose that $a_1, a_2, a_3, a_4, a_5, \ldots$ is a geometric sequence with common ratio 5. Suppose that you also know $a_2 = 20$. Determine each of a_1 and a_3, and then determine a_{20}.

Geometric sequence formula

If $a_1, a_2, a_3, a_4, a_5, \ldots$ is a geometric sequence with common ratio r, then a formula for the nth term is: $a_n = a_1 \cdot$_____

Exercise

Suppose that $a_1, a_2, a_3, a_4, a_5, \ldots$ is an geometric sequence. If $a_1 = 2,000,000$ and $a_4 = 31,250$, what is the common ratio? *Try using your formula above.*

Exercise

A study is run where participants will receive a guaranteed income for half of a year. Participants are offered two options:

- **Fixed monthly income**: Receive $5000 at the start of each month for 6 months.
- **Weekly-doubling income**: Receive 1 penny at the start of week 1. Each week thereafter (up through the 26th week), the person's total money is doubled. For example, at the start of week 2, they would have 2 pennies, and at the start of week 3, they would then have 4 pennies.

Discuss the pros and cons of each option. Make sure to compare how much money (in dollars) a participant would receive by the end of the half year. Which method would you prefer?