Patterns #6

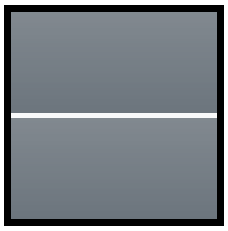
Introduction to recursive sequences

Exercise

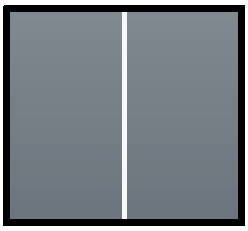
You are tiling walkways using 1 x 2 tiles (pictured below). When you use the tiles, you can lay them horizontally or vertically.

1 x 2 tile

We're curious how many different ways we could tile certain walkways. For example, if we want to tile a 2 x 2 walkway, there are *two* ways to do it (pictured below).

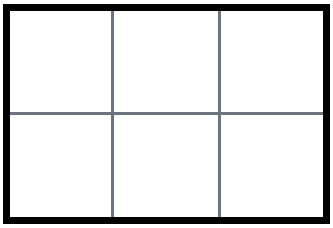


Tiling 1



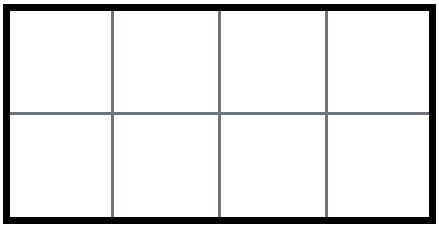
Tiling 2

1. How many ways are there to tile a 2 x 3 walkway? Sketch them out.



3 x 2 walkway

2. How about a 2 x 4 walkway?



4 x 2 walkway

3. How about a 2 x 5 walkway?

5 x 2 walkway

4. Let's define a sequence t_n where t_n tells us the number of ways to tile a 2 x n walkway. Use the work above to fill in the table below for $n = 2, 3, 4, 5$. Then look for a pattern in the numbers, and use it to complete the rest of table. What is the pattern?

n		2	3	4	5	6	7	8	9	10
t_n										

5. Can you explain the pattern you see in the numbers by connecting the ways to tile a 2 x 6 walkway with the ways to tile the smaller walkways? Generalize this to 2 x n.